

Minimal Model for Fast Scrambling

Ron Belyansky^{✉,*}, Przemyslaw Bienias, Yaroslav A. Kharkov, Alexey V. Gorshkov[✉], and Brian Swingle

*Joint Center for Quantum Information and Computer Science, NIST/University of Maryland,
College Park, Maryland 20742, USA and*

Joint Quantum Institute, NIST/University of Maryland, College Park, Maryland 20742, USA

 (Received 21 May 2020; accepted 24 August 2020; published 21 September 2020)

We study quantum information scrambling in spin models with both long-range all-to-all and short-range interactions. We argue that a simple global, spatially homogeneous interaction together with local chaotic dynamics is sufficient to give rise to fast scrambling, which describes the spread of quantum information over the entire system in a time that is logarithmic in the system size. This is illustrated in two tractable models: (1) a random circuit with Haar random local unitaries and a global interaction and (2) a classical model of globally coupled nonlinear oscillators. We use exact numerics to provide further evidence by studying the time evolution of an out-of-time-order correlator and entanglement entropy in spin chains of intermediate sizes. Our results pave the way towards experimental investigations of fast scrambling and aspects of quantum gravity with quantum simulators.

DOI: [10.1103/PhysRevLett.125.130601](https://doi.org/10.1103/PhysRevLett.125.130601)

Introduction.—The study of quantum information scrambling has recently attracted significant attention due to its relation to quantum chaos and thermalization of isolated many-body systems [1–3] as well as the dynamics of black holes [4–7]. Scrambling refers to the spread of initially local quantum information over the many-body degrees of freedom of the entire system, rendering it inaccessible to local measurements. Scrambling is also related to the Heisenberg dynamics of local operators, and can be probed via the squared commutator of two local and Hermitian operators W_1, V_r , at positions 1 and r , respectively,

$$\mathcal{C}(r, t) = -\frac{1}{2} \langle [W_1(t), V_r]^2 \rangle, \quad (1)$$

where $W_1(t)$ is the Heisenberg evolved operator. The growth of the squared commutator corresponds to $W_1(t)$ increasing in size and complexity, leading it to fail to commute with V_r . In a local quantum chaotic system, $\mathcal{C}(r, t)$ typically spreads ballistically, exhibiting rapid growth ahead of the wavefront and saturation behind, at late times [8–10].

Of particular interest are the so-called fast scramblers, systems where $\mathcal{C}(r, t)$ reaches $O(1)$ for all r in a time $t_s \propto \log(N)$, with N being the number of degrees of freedom. Among the best known examples are black holes, which are conjectured to be the fastest scramblers in nature [5–7, 11], as well as the Sachdev-Ye-Kitaev (SYK) [12, 13] model and other related holographic models [14–17].

Recent advances in the development of coherent quantum simulators have enabled the study of out-of-equilibrium dynamics of spin models with controllable interactions [18], making them ideal platforms to

experimentally study information scrambling. Several experiments have already been performed [19–24], probing scrambling in either local or nonchaotic systems. The experimental observation of fast scrambling remains challenging however, particularly because few systems are known to be fast scramblers, and those that are, like the SYK model, are highly nontrivial, involving random couplings and many-body interactions. Some recent proposals suggested that spin models with nonlocal interactions can exhibit fast scrambling [25–27], albeit with complicated and inhomogeneous interactions.

In this Letter, we argue that the simplest possible global interaction, together with chaotic dynamics, are sufficient to make a spin model fast scrambling. We consider spin-1/2 chains with Hamiltonians of the form

$$\mathcal{H} = \mathcal{H}_{\text{local}} - \frac{g}{\sqrt{N}} \sum_{i < j} Z_i Z_j, \quad (2)$$

where Z_i is the Pauli z operator acting on site i and $\mathcal{H}_{\text{local}}$ is a Hamiltonian with only local interactions that ensures that the full \mathcal{H} is chaotic. We note that such global interactions are ubiquitous in ultracold atoms in optical cavities [28–32], and also in ion traps [33–36].

We first show that this effect is generic, by studying two models, a random quantum circuit and a classical model, both designed to mimic the universal dynamics of Eq. (2). We then provide numerical evidence for fast scrambling for a particular time-independent quantum Hamiltonian. Finally, we discuss possible experimental realizations.

Random circuit model.—As a proof of principle, we consider a system of N spin-1/2 sites, with dynamics generated by a random quantum circuit (see Fig. 1) inspired

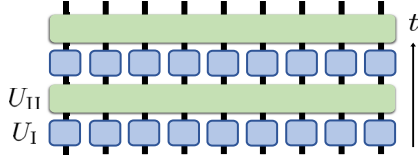


FIG. 1. Diagram of the random circuit. As given in Eq. (3), each blue square is an independent Haar-random unitary $U_{H,i}$ acting on site i , and the green rectangle is the global interaction U_{II} .

by the Hamiltonian in Eq. (2). While less physical than the Hamiltonian model, it has the advantage of being exactly solvable while providing intuition about generic many-body chaotic systems with similar features.

The time-evolution operator is $U(t) = (U_{II}U_I)^t$ where a single-time-step update consists of the two layers

$$U_I = \prod_{i=1}^N U_{H,i}, \quad U_{II} = e^{-i\frac{g}{2\sqrt{N}}\sum_{i<j}Z_iZ_j}, \quad (3)$$

where each $U_{H,i}$ is an independent Haar-random single-site unitary. The two layers in Eq. (3) are motivated by the two terms in Eq. (2), with the Haar-random unitaries replacing $\mathcal{H}_{\text{local}}$.

We are interested in the operator growth of an initially simple operator \mathcal{O} . At any point in time, the Heisenberg operator $\mathcal{O}(t) = U^\dagger(t)\mathcal{O}U(t)$ can be decomposed as $\mathcal{O}(t) = \sum_{\mathcal{S}} a_{\mathcal{S}}(t)\mathcal{S}$, where \mathcal{S} is a string composed of the Pauli matrices and the identity, forming a basis for $SU(2^N)$. As in random brickwork models [37,38] and random Brownian models [9], the Haar-averaged probabilities $\langle a_{\mathcal{S}}^2(t) \rangle$, encoding the time evolution of $\mathcal{O}(t)$, themselves obey a linear equation

$$\langle a_{\mathcal{S}}^2(t+1) \rangle = \sum_{\mathcal{S}'} W_{\mathcal{S},\mathcal{S}'} \langle a_{\mathcal{S}'}^2(t) \rangle. \quad (4)$$

Here, $W_{\mathcal{S},\mathcal{S}'}$ is a $4^N \times 4^N$ stochastic matrix describing a fictitious Markov process [39,40]. The average probabilities $\langle a_{\mathcal{S}}^2(t) \rangle$ fully determine the growth of the average of $\mathcal{C}(t)$ in Eq. (1) [see Supplemental Material (SM) [41]]. Because of the Haar unitaries and the simple uniform interaction in Eq. (3), $W_{\mathcal{S},\mathcal{S}'}$ is highly degenerate and only depends on the total weights of the strings $\mathcal{S}, \mathcal{S}'$, counting the number of nonidentity operators, i.e., $w(\mathcal{S}) = \sum_i (1 - \delta_{\mathcal{S},1})$, and on the number of sites where both \mathcal{S} and \mathcal{S}' are nonidentity, i.e., $v(\mathcal{S}, \mathcal{S}') = \sum_i (1 - \delta_{\mathcal{S},1})(1 - \delta_{\mathcal{S}',1})$, and is given by (see SM for derivation [41]) [43]

$$W(w, w', v) = \left(\frac{1}{3}\right)^{w+w'} \sum_{k=0}^v \binom{v}{k} \sum_{l=0}^k \binom{k}{l} \times \left[\cos^2\left(\frac{2l-k}{\sqrt{N}}g\right) \right]^{N-k-(w+w'-2v)} \left[\sin^2\left(\frac{2l-k}{\sqrt{N}}g\right) \right]^{w+w'-2v}. \quad (5)$$

If we further assume that \mathcal{O} starts out as a single site operator on site 1, then throughout the evolution, $\langle a_{\mathcal{S}}^2(t) \rangle$ only depend on the total operator weight w , and the weight on site 1, which we denote by $w_1 \in \{0, 1\}$. We thus introduce the operator weight probability h_t at time t ,

$$h_t(w, w_1) = \langle a_{\mathcal{S}}^2(t) \rangle 3^w \binom{N-1}{w-w_1}, \quad (6)$$

which gives the probability of $\mathcal{O}(t)$ having total weight w and weight w_1 on site 1.

The time evolution of $h_t(w, w_1)$ is given by the master equation

$$h_{t+1}(w, w_1) = \sum_{w'_1=0,1} \sum_{w'=w'_1}^{N-1+w'_1} \mathcal{R}(w, w_1, w', w'_1) h_t(w', w'_1), \quad (7)$$

where the $2N \times 2N$ matrix \mathcal{R} is

$$\mathcal{R}(w, w_1, w', w'_1) = 3^w \sum_{m=0}^{\min\{w-w_1, w'-w'_1\}} \binom{w'-w'_1}{m} \times \binom{N-1-w'+w'_1}{w-w_1-m} W(w, w', m+w_1w'_1). \quad (8)$$

The transition matrix \mathcal{R} , scaling only linearly with N , allows us to efficiently simulate the dynamics for large system sizes (see Fig. 2).

To proceed analytically, we Taylor expand Eq. (5) to leading order in g , which gives rise to a closed master equation for the total operator weight probability $h_t(w) \equiv h_t(w, 0) + h_t(w, 1)$,

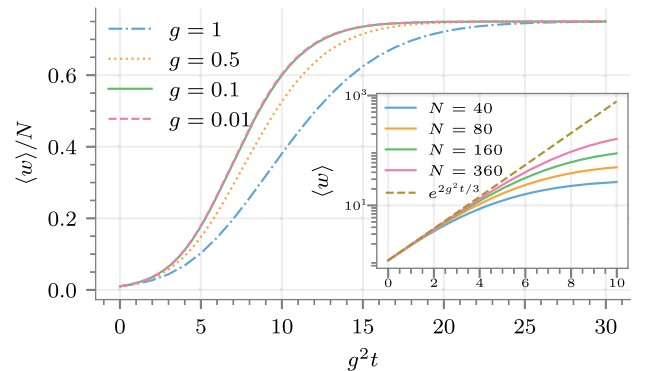


FIG. 2. Normalized mean operator weight $\langle w(t) \rangle / N = (1/N) \sum_w w h_t(w)$ as a function of time for different g and $N = 100$, computed using Eq. (7). For small enough g , all the curves collapse to a single curve as a function of $g^2 t$, as implied by Eq. (10). The inset shows the initial exponential increase of $\langle w(t) \rangle$ for different system sizes N and $g = 0.1$.

$$\begin{aligned} \frac{h_{r+1}(w) - h_r(w)}{g^2} &= \frac{2w}{9N}(1 - 3N + 2w)h_r(w) \\ &+ \frac{2w(w+1)}{9N}h_r(w+1) \\ &+ \frac{N-w+1}{3N}2(w-1)h_r(w-1), \end{aligned} \quad (9)$$

which is similar to random Brownian models [9,44] and shows that, at $O(g^2)$, w can change by at most ± 1 in a single step. Assuming that $h(w, t)$ varies slowly with respect to $g^2 t$ and w , we can approximate the above equation by a Fokker-Planck equation (rescaling time $\tau = g^2 t$)

$$\partial_\tau h(w, \tau) = -\partial_w [D_1(w)h(w, \tau)] + \partial_w^2 [D_2(w)h(w, \tau)], \quad (10)$$

where the drift and diffusion coefficients are [dropping higher order terms $O(1/N, w/N)$]

$$D_1(w) = \frac{2}{3} \left(w - \frac{4w^2}{3N} \right), \quad D_2(w) = \frac{w}{3} - \frac{2w^2}{9N}. \quad (11)$$

This equation describes the rapid growth of an initially localized distribution, followed by a broadening and finally saturation (see Fig. 2 and SM [41] for more details). At early time, the $\frac{2}{3}w$ term in the drift coefficient dominates, giving rise to exponential growth of the mean operator weight $\langle w(t) \rangle \sim e^{2g^2 t/3}$, which agrees with the full numerical solution of the master equation, as can be seen in Fig. 2. The mean weight is related to the infinite-temperature squared-commutator in Eq. (1) (averaged over different circuits) via $\langle \mathcal{C}(t) \rangle = \frac{4}{3} \langle w(t) \rangle / N$ [41]. Since $\langle w(t) \rangle$ grows exponentially with time, $\langle w(t) \rangle$ reaches $O(N)$ and $\langle \mathcal{C}(t) \rangle$ reaches $O(1)$ when $t = (3/2g^2) \log(N)$, thus establishing that this model is fast scrambling. Note that the $1/\sqrt{N}$ normalization in Eqs. (2) and (3) is crucial. Had we chosen instead $1/N (g \rightarrow g/\sqrt{N})$, the Lyapunov exponent would have been $2g^2/3N$ and the scrambling time would have been $t \sim N \log(N)$.

Classical model.—Let us now consider a different setting that also allows us to probe the basic timescales involved, and shows that randomness is not required. A convenient tractable choice is a classical model consisting of globally coupled nonlinear oscillators. Note that the analogs of out-of-time-order correlators (OTOCs) have been studied in a variety of classical models [26,45–49] and have been shown to capture the scrambling dynamics of quantum models like the SYK model [50–52].

Consider a $2N$ -dimensional phase space with coordinates q_r (positions) and p_r (momenta) for $r = 1, \dots, N$ with canonical structure specified by the Poisson brackets $\{q_r, p_s\}_{\text{PB}} = \delta_{rs}$. The Hamiltonian is $\mathcal{H}_c = K + V_2 + V_4$ where

$$K = \sum_{r=1}^N \frac{p_r^2}{2}, \quad V_4 = \frac{\Omega_3^2}{4} \sum_{r=1}^N q_r^4, \quad (12)$$

$$V_2 = \frac{\Omega_1^2}{2} \sum_{r=1}^{N-1} (q_{r+1} - q_r)^2 + \frac{\Omega_2^2}{2\sqrt{N}} \left(\sum_{r=1}^N q_r \right)^2. \quad (13)$$

The timescales for the growth of perturbations under \mathcal{H}_c dynamics may be understood in two stages. First, $K + V_2$ can be solved exactly; this combination of terms provides the nonlocality. The remaining V_4 term renders the dynamics chaotic, provided Ω_3 is large enough. The dynamics of $K + V_2$ causes a localized perturbation to spread to every oscillator with nonlocal amplitude $1/N$ in a time of order $1/N^{1/4}\Omega_2$. Then conventional local chaos can amplify this $1/N$ -sized perturbation to order-one size in a time of order $\lambda^{-1} \ln N$, where λ is some typical Lyapunov exponent.

At the quadratic level, the uniform mode, $Q = (1/N) \sum_r q_r$, is decoupled from the remaining modes of the chain. Hence, the propagation of any perturbation is a superposition of the motion due to the local Ω_1 terms and the special dynamics of the uniform mode. Since the local terms cannot induce nonlocal perturbations, we may focus on the dynamics of the uniform mode. The uniform mode's equation of motion is $(d^2 Q/dt^2) = -\sqrt{N}\Omega_2^2 Q$ with solution

$$Q(t) = Q(0) \cos(N^{1/4}\Omega_2 t) + \frac{dQ(0)}{dt} \frac{1}{N^{1/4}\Omega_2} \sin(N^{1/4}\Omega_2 t). \quad (14)$$

A localized perturbation on site 1 with zero initial time derivative can be written as $\delta \vec{q}(0) = \epsilon([\hat{\mathbf{e}}_1 - \hat{\mathbf{u}}_0] + \hat{\mathbf{u}}_0)$, where $\hat{\mathbf{u}}_0 = [1, \dots, 1]^T/N$ represents the uniform mode, $\hat{\mathbf{e}}_1 = [1, 0, \dots, 0]^T$, and $\hat{\mathbf{e}}_1 - \hat{\mathbf{u}}_0$ is orthogonal to the uniform mode. The orthogonal mode evolves in a local fashion, hence $\delta \vec{q}(t) = \epsilon[\text{local piece} + \hat{\mathbf{u}}_0 \cos(N^{1/4}\Omega_2 t)]$. For oscillators far from the initial local perturbation, the dynamics is given by

$$\delta q_{r \gg 1}(t) = \frac{\epsilon}{N} \left[\cos\left(N^{1/4}\Omega_2 t\right) - 1 \right]. \quad (15)$$

Thus, after a time $\pi/N^{1/4}\Omega_2$, any localized perturbation has spread to distant sites with amplitude ϵ/N .

The inclusion of V_4 renders the equations of motion nonlinear and the system chaotic in at least part of the phase space. We leave a detailed study of the classical chaotic dynamics of this model to the future, but as can be seen in Fig. 3, a numerical solution of the equations of motion displays sensitivity to initial conditions.

The precise protocol is as follows. We compare the dynamics of two configurations, $\vec{q}^{(1)}$ and $\vec{q}^{(2)}$, averaged over many initial conditions. The initial condition of configuration one has each oscillator start at rest from a

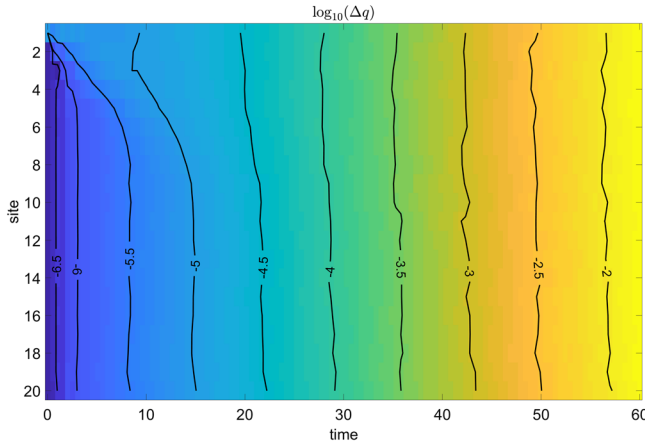


FIG. 3. $\log_{10} \Delta q_r(t)$ for $N = 20$, $\epsilon = 10^{-5}$, $\Omega_1 = \Omega_2 = 1$, and $\Omega_3 = 2$. The labeled black lines are contours of constant $\log_{10} \Delta q$. Early time ballistic growth is visible in the upper left corner while at later times the system exhibits spatially uniform exponential growth in time.

random amplitude drawn uniformly and independently from $[-1, 1]$. Configuration two is identical to configuration one except that $q_1^{(2)}(0) = q_1^{(1)}(0) + \epsilon$ for $\epsilon = 10^{-5}$. Both configurations are evolved in time and the difference $\Delta q_r(t) = |q_r^{(2)}(t) - q_r^{(1)}(t)|$ is computed and averaged over 4000 different initial conditions. Figure 3 shows this average of Δq_r for $N = 20$ with $\Omega_1 = 1$, $\Omega_2 = 1$, and $\Omega_3 = 2$. Because the system can generate an ϵ/N -sized perturbation on all sites in a short time, the subsequent uniform exponential growth implies that any local perturbation will become order one on all sites after a time $\sim \lambda^{-1} \log(N/\epsilon)$.

The above analysis corresponds to the classical limit of coupled quantum oscillators where some effective dimensionless Planck's constant vanishes, $\hbar_{\text{eff}} \rightarrow 0$. In the opposite limit of large N at fixed \hbar_{eff} , the dynamics of quantum OTOCs can be obtained from the corresponding classical Lyapunov growth up to a timescale of order $\log(1/\hbar_{\text{eff}}) \ll \log N$. At later times, one needs to consider fully quantum local dynamics. If one imagines breaking the system up into local clusters and if each cluster can be viewed as a quantum chaotic system with random-matrix-like energy levels, a dynamical system not unlike the random circuit model above is obtained.

Chaos and level statistics.—Having established fast scrambling in both the random circuit and the classical model, we now return to the quantum spin model of Eq. (2). We first examine whether such a model is chaotic, which is a necessary condition for it being fast scrambling. For the local Hamiltonian part, we consider the mixed-field Ising chain

$$\mathcal{H}_{\text{local}} = -J \sum_i Z_i Z_{i+1} - h_x \sum_i X_i - h_z \sum_i Z_i. \quad (16)$$

A standard approach to identify a transition from integrability to quantum chaos is based on a comparison of energy-level-spacing statistics with Poisson and Wigner-Dyson distributions. Another convenient metric is the average ratio of consecutive level spacings [53] $\langle r \rangle$, where $r = \min(r_n, 1/r_n)$, $r_n = \delta_n/\delta_{n-1}$, $\delta_n = E_n - E_{n-1}$, and E_n are the eigenvalues ordered such that $E_n \geq E_{n-1}$.

As was already suggested in Ref. [54] for a similar model, we find that the longitudinal field is unnecessary, and the full system can have Wigner-Dyson statistics even for $h_z = 0$, in which case $\mathcal{H}_{\text{local}}$ is integrable. The resulting Hamiltonian reads

$$\mathcal{H} = -J \sum_i Z_i Z_{i+1} - h_x \sum_i X_i - \frac{g}{\sqrt{N}} \sum_{i < j} Z_i Z_j. \quad (17)$$

The average adjacent-level-spacing ratio changes from $\langle r \rangle_{\text{Pois}} \approx 0.38$ for Poisson level statistics to $\langle r \rangle_{\text{GOE}} \approx 0.53$ for Wigner-Dyson level statistics in the Gaussian orthogonal ensemble (GOE) [53]. In the vicinity of $g \rightarrow 0$, $\langle r \rangle$ (see Fig. 4) shows proximity to Poisson statistics, while, for $|g| \gtrsim 0.25$, the level statistics agree with those of the GOE.

Out-of-time-order correlator and entanglement growth.—We now study the dynamics of an OTOC and entanglement entropy in the spin chain. We consider the following OTOC:

$$F(r, t) = \Re[\langle Z_1(t) Z_r Z_1(t) Z_r \rangle], \quad (18)$$

which is related to Eq. (1) by $\mathcal{C}(r, t) = 1 - F(r, t)$. The expectation value is evaluated in a Haar-random pure state, which approximates the infinite-temperature OTOC, but enables us to reach larger system sizes [55].

In Fig. 5(a), we show the OTOC for an open chain of $N = 20$ spins for both the local model, governed by $\mathcal{H}_{\text{local}}$ only, and the nonlocal model in Eq. (17), which includes the global interaction. In the local case, the OTOC spreads ballistically, forming a linear light cone. In contrast, in the nonlocal case, the spreading is superballistic and $F(r \gg 1, t)$ is approximately independent of r , as expected

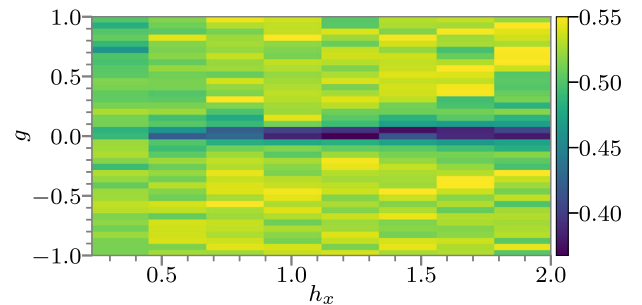


FIG. 4. Average adjacent-level-spacing ratio $\langle r \rangle$ for the model in Eq. (17) with $J = 1$. Data corresponds to a system of $N = 15$ spins with periodic boundary conditions for fixed momentum and Z -reflection symmetry blocks of the Hamiltonian.

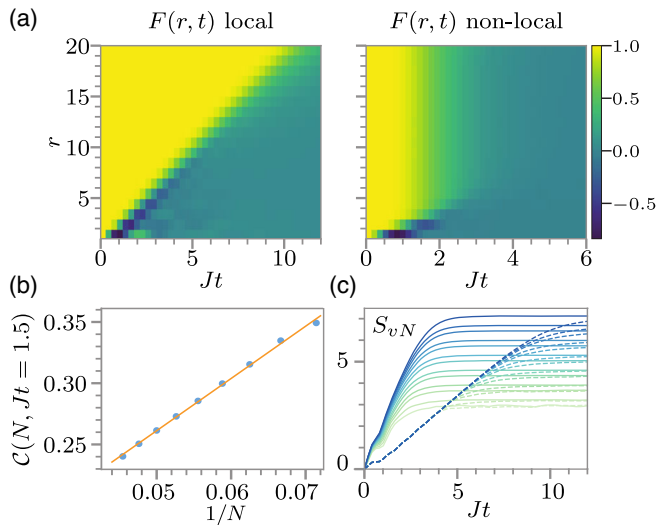


FIG. 5. (a) Time evolution of the OTOC for the (left) local and (right) nonlocal models. (b) $1 - F(N, t)$ after a fixed evolution time for the nonlocal model (for different system sizes N), showing a linear dependence on $1/N$. The orange line is a linear fit. (c) Half-cut entanglement-entropy growth starting from the $+\hat{y}$ state for local (dashed lines) and nonlocal (solid lines) models. The color indicates the system size, starting from $N = 10$ (light green) until $N = 22$ (dark blue). For all plots, $J = 1, h_x = 1.05$, and $h_z = 0.5, g = 0$ ($h_z = 0, g = -1$) for the local (nonlocal) models.

for a fast scrambler. As we discussed in the context of the classical model, a necessary condition for fast scrambling is that, before the onset of exponential growth, the decay of correlations with N should be at most algebraic ($\mathcal{C} \propto N^{-\alpha}$) and not exponential. In Fig. 5(b), we verify that this is the case for the nonlocal model, showing that $\mathcal{C} \propto N^{-1}$ between the two ends of the chain after a fixed time.

Figure 5(c) shows the half-cut entanglement entropy following a quench starting from the $+\hat{y}$ state for both models. For the local model, the entanglement grows linearly in time before saturating, whereas the nonlocal model shows a significant speed up. Moreover, in the nonlocal model, the growth rate clearly increases with the system size, further supporting our claim.

Experimental realization.—The Hamiltonian in Eq. (17), and many variations of it, can be experimentally realized in a variety of platforms. A natural realization is with Rydberg dressing of neutral atoms [56–59]. The spin can be encoded in two ground states with one of them dressed to two Rydberg states such that one of the Rydberg states leads to all-to-all interactions and the second to nearest-neighbor interactions. Other similar spin models can be realized with cavity-QED setups, using photon-mediated all-to-all interactions [28,31,60,61] of the XX or XXZ -Heisenberg form [25,27] together with nearest-neighbor interactions achieved by Rydberg dressing one of the ground states [62,63]. Other possibilities include a chain of coupled superconducting qubits, with all-to-all flip-flop interactions

mediated via a common bus [64–66] or trapped ions [33–36,67].

Conclusion and outlook.—In this Letter, we argued that a single global interaction together with local chaotic dynamics is sufficient to give rise to fast scrambling. While fast scrambling is intrinsically difficult to study numerically, our numerical evidence, together with the semiclassical analysis and the exactly solvable random circuit, provide a compelling argument in favor of our claim. Our models do not require disordered or inhomogeneous couplings and are within reach of current state-of-the-art quantum simulators. Thus, an experimental implementation of the spin model could test our claims on much larger system sizes, something that may very well be impossible to do on a classical computer. This can pave the way towards experimental investigations of aspects of quantum gravity.

Future theoretical work may include a more systematic analysis of the N dependence of various timescales, e.g., for entanglement growth, and of the behavior of the OTOC at low temperatures. It is also interesting to investigate whether similar conclusions can be reached without perfectly uniform global interactions, for example with power-law decaying interactions.

We thank A. Lucas for useful discussions, especially regarding the normalization of the global interactions. We acknowledge discussions with S. Xu at early stages of this work and P. Titum for pointing out the Hamiltonian [Eq. (17)] to us. R. B., P. B., Y. A. K., and A. V. G. acknowledge funding by the NSF PFCQC program, DoE ASCR Quantum Testbed Pathfinder program (Award No. DE-SC0019040), DoE BES Award No. DE-SC0019449, DoE ASCR Accelerated Research in Quantum Computing program (Award No. DE-SC0020312), AFOSR Multidisciplinary University Research Initiative (MURI), AFOSR, ARO MURI, NSF PFC at JQI, and ARL CDQI. R. B. acknowledges support of NSERC and FRQNT of Canada. B. G. S. is supported in part by the U.S. Department of Energy, Office of Science, Office of High Energy Physics QuantISED Award de-sc0019380.

Note added.—We would like to draw the reader’s attention to two related parallel works which appeared recently: by Li, Choudhury, and Liu [68], on fast scrambling with similar spin models; and by Yin and Lucas [69], on lower bounds of the scrambling time in similar spin models.

*rbelyans@umd.edu

- [1] J. M. Deutsch, Quantum statistical mechanics in a closed system, *Phys. Rev. A* **43**, 2046 (1991).
- [2] M. Srednicki, Chaos and quantum thermalization, *Phys. Rev. E* **50**, 888 (1994).

- [3] M. Rigol, V. Dunjko, and M. Olshanii, Thermalization and its mechanism for generic isolated quantum systems, *Nature (London)* **452**, 854 (2008).
- [4] P. Hayden and J. Preskill, Black holes as mirrors: Quantum information in random subsystems, *J. High Energy Phys.* **09** (2007) 120.
- [5] N. Lashkari, D. Stanford, M. Hastings, T. Osborne, and P. Hayden, Towards the fast scrambling conjecture, *J. High Energy Phys.* **4** (2013) 022.
- [6] S. H. Shenker and D. Stanford, Black holes and the butterfly effect, *J. High Energy Phys.* **3** (2014) 067.
- [7] J. Maldacena, S. H. Shenker, and D. Stanford, A bound on chaos, *J. High Energy Phys.* **8** (2016) 106.
- [8] A. A. Patel, D. Chowdhury, S. Sachdev, and B. Swingle, Quantum Butterfly Effect in Weakly Interacting Diffusive Metals, *Phys. Rev. X* **7**, 031047 (2017).
- [9] S. Xu and B. Swingle, Locality, Quantum Fluctuations, and Scrambling, *Phys. Rev. X* **9**, 031048 (2019).
- [10] S. Xu and B. Swingle, Accessing scrambling using matrix product operators, *Nat. Phys.* **16**, 199 (2020).
- [11] Y. Sekino and L. Susskind, Fast scramblers, *J. High Energy Phys.* **10** (2008) 065.
- [12] S. Sachdev and J. Ye, Gapless Spin-Fluid Ground State in a Random Quantum Heisenberg Magnet, *Phys. Rev. Lett.* **70**, 3339 (1993).
- [13] A. Kitaev, A simple model of quantum holography, in *KITP Program: Entanglement in Strongly-Correlated Quantum Matter* (2015), <http://online.kitp.ucsb.edu/online/entangled15/kitaev/>.
- [14] I. Danshita, M. Hanada, and M. Tezuka, Creating and probing the Sachdev-Ye-Kitaev model with ultracold gases: Towards experimental studies of quantum gravity, *Prog. Theor. Exp. Phys.* **2017**, 83 (2017).
- [15] A. Chen, R. Ilan, F. de Juan, D. I. Pikulin, and M. Franz, Quantum Holography in a Graphene Flake with an Irregular Boundary, *Phys. Rev. Lett.* **121**, 036403 (2018).
- [16] A. Chew, A. Essin, and J. Alicea, Approximating the Sachdev-Ye-Kitaev model with Majorana wires, *Phys. Rev. B* **96**, 121119(R) (2017).
- [17] Y. Gu, X.-L. Qi, and D. Stanford, Local criticality, diffusion and chaos in generalized Sachdev-Ye-Kitaev models, *J. High Energy Phys.* **05** (2017) 125.
- [18] J. I. Cirac and P. Zoller, Goals and opportunities in quantum simulation, *Nat. Phys.* **8**, 264 (2012).
- [19] J. Li, R. Fan, H. Wang, B. Ye, B. Zeng, H. Zhai, X. Peng, and J. Du, Measuring Out-of-Time-Order Correlators on a Nuclear Magnetic Resonance Quantum Simulator, *Phys. Rev. X* **7**, 031011 (2017).
- [20] M. Gärtner, J. G. Bohnet, A. Safavi-Naini, M. L. Wall, J. J. Bollinger, and A. M. Rey, Measuring out-of-time-order correlations and multiple quantum spectra in a trapped-ion quantum magnet, *Nat. Phys.* **13**, 781 (2017).
- [21] K. X. Wei, C. Ramanathan, and P. Cappellaro, Exploring Localization in Nuclear Spin Chains, *Phys. Rev. Lett.* **120**, 070501 (2018).
- [22] K. A. Landsman, C. Figgatt, T. Schuster, N. M. Linke, B. Yoshida, N. Y. Yao, and C. Monroe, Verified quantum information scrambling, *Nature (London)* **567**, 61 (2019).
- [23] M. K. Joshi, A. Elben, B. Vermersch, T. Brydges, C. Maier, P. Zoller, R. Blatt, and C. F. Roos, Quantum information scrambling in a trapped-ion quantum simulator with tunable range interactions, *Phys. Rev. Lett.* **124**, 240505 (2020).
- [24] M. S. Blok, V. V. Ramasesh, T. Schuster, K. O'Brien, J. M. Kreikebaum, D. Dahlen, A. Morvan, B. Yoshida, N. Y. Yao, and I. Siddiqi, Quantum Information Scrambling in a Superconducting Qutrit Processor, [arXiv:2003.03307](https://arxiv.org/abs/2003.03307).
- [25] B. Swingle, G. Bentsen, M. Schleier-Smith, and P. Hayden, Measuring the scrambling of quantum information, *Phys. Rev. A* **94**, 040302(R) (2016).
- [26] J. Marino and A. M. Rey, Cavity-QED simulator of slow and fast scrambling, *Phys. Rev. A* **99**, 051803(R) (2019).
- [27] G. Bentsen, T. Hashizume, A. S. Buyskikh, E. J. Davis, A. J. Daley, S. S. Gubser, and M. Schleier-Smith, Treelike Interactions and Fast Scrambling with Cold Atoms, *Phys. Rev. Lett.* **123**, 130601 (2019).
- [28] A. S. Sørensen and K. Mølmer, Entangling atoms in bad cavities, *Phys. Rev. A* **66**, 022314 (2002).
- [29] S. Chaudhury, S. Merkel, T. Herr, A. Silberfarb, I. H. Deutsch, and P. S. Jessen, Quantum Control of the Hyperfine Spin of a Cs Atom Ensemble, *Phys. Rev. Lett.* **99**, 163002 (2007).
- [30] T. Fernholz, H. Krauter, K. Jensen, J. F. Sherson, A. S. Sørensen, and E. S. Polzik, Spin Squeezing of Atomic Ensembles Via Nuclear-Electronic Spin Entanglement, *Phys. Rev. Lett.* **101**, 073601 (2008).
- [31] I. D. Leroux, M. H. Schleier-Smith, and V. Vuletić, Implementation of Cavity Squeezing of a Collective Atomic Spin, *Phys. Rev. Lett.* **104**, 073602 (2010).
- [32] M. H. Schleier-Smith, I. D. Leroux, and V. Vuletić, Squeezing the collective spin of a dilute atomic ensemble by cavity feedback, *Phys. Rev. A* **81**, 021804(R) (2010).
- [33] B. C. Sawyer, J. W. Britton, A. C. Keith, C.-C. J. Wang, J. K. Freericks, H. Uys, M. J. Biercuk, and J. J. Bollinger, Spectroscopy and Thermometry of Drumhead Modes in a Mesoscopic Trapped-Ion Crystal Using Entanglement, *Phys. Rev. Lett.* **108**, 213003 (2012).
- [34] J. W. Britton, B. C. Sawyer, A. C. Keith, C.-C. J. Wang, J. K. Freericks, H. Uys, M. J. Biercuk, and J. J. Bollinger, Engineered two-dimensional Ising interactions in a trapped-ion quantum simulator with hundreds of spins, *Nature (London)* **484**, 489 (2012).
- [35] C.-C. J. Wang, A. C. Keith, and J. K. Freericks, Phonon-mediated quantum spin simulator employing a planar ionic crystal in a Penning trap, *Phys. Rev. A* **87**, 013422 (2013).
- [36] J. G. Bohnet, B. C. Sawyer, J. W. Britton, M. L. Wall, A. M. Rey, M. Foss-Feig, and J. J. Bollinger, Quantum spin dynamics and entanglement generation with hundreds of trapped ions, *Science* **352**, 1297 (2016).
- [37] A. Nahum, S. Vijay, and J. Haah, Operator Spreading in Random Unitary Circuits, *Phys. Rev. X* **8**, 021014 (2018).
- [38] C. W. von Keyserlingk, T. Rakovszky, F. Pollmann, and S. L. Sondhi, Operator Hydrodynamics, OTOCs, and Entanglement Growth in Systems Without Conservation Laws, *Phys. Rev. X* **8**, 021013 (2018).
- [39] O. C. O. Dahlsten, R. Oliveira, and M. B. Plenio, The emergence of typical entanglement in two-party random processes, *J. Phys. A* **40**, 8081 (2007).

- [40] M. Žnidarič, Exact convergence times for generation of random bipartite entanglement, *Phys. Rev. A* **78**, 032324 (2008).
- [41] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.125.130601> for additional details concerning the random circuit including the derivation, which includes Ref. [42].
- [42] L. Zhang, Matrix integrals over unitary groups: An application of Schur-Weyl duality, [arXiv:1408.3782](https://arxiv.org/abs/1408.3782).
- [43] The term with $2l = k, w + w' - 2v = 0$ is assumed to be $0^0 = 1$. See Ref. [41].
- [44] T. Zhou and X. Chen, Operator dynamics in a Brownian quantum circuit, *Phys. Rev. E* **99**, 052212 (2019).
- [45] E. B. Rozenbaum, S. Ganeshan, and V. Galitski, Lyapunov Exponent and Out-of-Time-Ordered Correlator's Growth Rate in a Chaotic System, *Phys. Rev. Lett.* **118**, 086801 (2017).
- [46] T. Bilitewski, S. Bhattacharjee, and R. Moessner, Temperature Dependence of the Butterfly Effect in a Classical Many-Body System, *Phys. Rev. Lett.* **121**, 250602 (2018).
- [47] J. Chávez-Carlos, B. López-del Carpio, M. A. Bastarrachea-Magnani, P. Stránský, S. Lerma-Hernández, L. F. Santos, and J. G. Hirsch, Quantum and Classical Lyapunov Exponents in Atom-Field Interaction Systems, *Phys. Rev. Lett.* **122**, 024101 (2019).
- [48] R. A. Jalabert, I. García-Mata, and D. A. Wisniacki, Semiclassical theory of out-of-time-order correlators for low-dimensional classically chaotic systems, *Phys. Rev. E* **98**, 062218 (2018).
- [49] B. Yan, L. Cincio, and W. H. Zurek, Information Scrambling and Loschmidt Echo, *Phys. Rev. Lett.* **124**, 160603 (2020).
- [50] J. Kurchan, Quantum bound to chaos and the semiclassical limit, *J. Stat. Phys.* **171**, 965 (2018).
- [51] M. Schmitt, D. Sels, S. Kehrein, and A. Polkovnikov, Semiclassical echo dynamics in the Sachdev-Ye-Kitaev model, *Phys. Rev. B* **99**, 134301 (2019).
- [52] T. Scaffidi and E. Altman, Chaos in a classical limit of the Sachdev-Ye-Kitaev model, *Phys. Rev. B* **100**, 155128 (2019).
- [53] Y. Y. Atas, E. Bogomolny, O. Giraud, and G. Roux, Distribution of the Ratio of Consecutive Level Spacings in Random Matrix Ensembles, *Phys. Rev. Lett.* **110**, 084101 (2013).
- [54] A. Lerose, J. Marino, B. Žunkovič, A. Gambassi, and A. Silva, Chaotic Dynamical Ferromagnetic Phase Induced by Nonequilibrium Quantum Fluctuations, *Phys. Rev. Lett.* **120**, 130603 (2018).
- [55] D. J. Luitz and Y. B. Lev, Information propagation in isolated quantum systems, *Phys. Rev. B* **96**, 020406(R) (2017).
- [56] J. Honer, H. Weimer, T. Pfau, and H. P. Büchler, Collective Many-Body Interaction in Rydberg Dressed Atoms, *Phys. Rev. Lett.* **105**, 160404 (2010).
- [57] T. Graß, P. Bienias, M. J. Gullans, R. Lundgren, J. Maciejko, and A. V. Gorshkov, Fractional Quantum Hall Phases of Bosons with Tunable Interactions: From the Laughlin Liquid to a Fractional Wigner Crystal, *Phys. Rev. Lett.* **121**, 253403 (2018).
- [58] G. Pupillo, A. Micheli, M. Boninsegni, I. Lesanovsky, and P. Zoller, Strongly Correlated Gases of Rydberg-Dressed Atoms: Quantum and Classical Dynamics, *Phys. Rev. Lett.* **104**, 223002 (2010).
- [59] A. W. Glaetzle, R. Nath, B. Zhao, G. Pupillo, and P. Zoller, Driven-dissipative dynamics of a strongly interacting Rydberg gas, *Phys. Rev. A* **86**, 043403 (2012).
- [60] S. Gopalakrishnan, B. L. Lev, and P. M. Goldbart, Frustration and Glassiness in Spin Models with Cavity-Mediated Interactions, *Phys. Rev. Lett.* **107**, 277201 (2011).
- [61] P. Strack and S. Sachdev, Dicke Quantum Spin Glass of Atoms and Photons, *Phys. Rev. Lett.* **107**, 277202 (2011).
- [62] J. Gelhausen, M. Buchhold, A. Rosch, and P. Strack, Quantum-optical magnets with competing short- and long-range interactions: Rydberg-dressed spin lattice in an optical cavity, *SciPost Phys.* **1**, 004 (2016).
- [63] B. Zhu, J. Marino, N. Y. Yao, M. D. Lukin, and E. A. Demler, Dicke time crystals in driven-dissipative quantum many-body systems, *New J. Phys.* **21**, 073028 (2019).
- [64] J. Majer, J. M. Chow, J. M. Gambetta, J. Koch, B. R. Johnson, J. A. Schreier, L. Frunzio, D. I. Schuster, A. A. Houck, A. Wallraff, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Coupling superconducting qubits via a cavity bus, *Nature (London)* **449**, 443 (2007).
- [65] G. Zhu, M. Hafezi, and T. Grover, Measurement of many-body chaos using a quantum clock, *Phys. Rev. A* **94**, 062329 (2016).
- [66] T. Onodera, E. Ng, and P. L. McMahon, A quantum annealer with fully programmable all-to-all coupling via Floquet engineering, *npj Quantum Inf.* **6**, 48 (2020).
- [67] S. Genway, W. Li, C. Ates, B. P. Lanyon, and I. Lesanovsky, Generalized Dicke Nonequilibrium Dynamics in Trapped Ions, *Phys. Rev. Lett.* **112**, 023603 (2014).
- [68] Z. Li, S. Choudhury, and W. V. Liu, Fast scrambling without appealing to holographic duality, [arXiv:2004.11269](https://arxiv.org/abs/2004.11269).
- [69] C. Yin and A. Lucas, Bound on quantum scrambling with all-to-all interactions, *Phys. Rev. A* **102**, 022402 (2020).