Fast Quantum State Transfer and Entanglement Renormalization Using Long-Range Interactions

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In short-range interacting systems, the speed at which entanglement can be established between two separated points is limited by a constant Lieb-Robinson velocity. Long-range interacting systems are capable of faster entanglement generation, but the degree of the speedup possible is an open question. In this Letter, we present a protocol capable of transferring a quantum state across a distance L in d dimensions using long-range interactions with a strength bounded by $1/r^\alpha$. If $\alpha < d$, the state transfer time is asymptotically independent of L; if $\alpha = d$, the time scales logarithmically with the distance L; if $d < \alpha < d + 1$, the transfer occurs in a time proportional to $L^{\alpha-d}$; and if $\alpha \geq d + 1$, it occurs in a time proportional to L. We then use this protocol to upper bound the time required to create a state specified by a multiscale entanglement renormalization ansatz (MERA) tensor network and show that if the linear size of the MERA state is L, then it can be created in a time that scales with L identically to the state transfer up to logarithmic corrections. This protocol realizes an exponential speedup in cases of $\alpha = d$, which could be useful in creating large entangled states for dipole-dipole ($1/r^3$) interactions in three dimensions.

Entanglement generation in a quantum system is limited, even in a nonrelativistic setting, by the available interactions. In a lattice system with short-range interactions, Lieb and Robinson showed that there exists a linear light cone defined by a speed proportional to both the interaction range and strength [1]. Suppose two operators A and B are supported on single sites separated by a distance r. Then the Lieb-Robinson bound states that, after time t, $\|[A(t), B]\| \leq c\|A\|\|B\|e^{vt-r}$, where c is a constant, v is another constant known as the Lieb-Robinson velocity, and $\|\cdot\|$ represents the operator norm. If a system initially in a product state begins evolving under a short-range Hamiltonian, correlations decrease exponentially outside of the causal cone defined by $r = vt$ [2–4]. However, in physical systems including polar molecules [5–7], Rydberg atoms [8,9], or trapped ions [10,11], the interactions fall off with distance r as a power law $1/r^\alpha$. For these interactions, generalizations of the Lieb-Robinson bound are known, but they may not be tight [12–14]. In addition, for sufficiently long-ranged interactions, the causal region may even encompass infinite space at a finite time, signaling a breakdown of emergent locality [15–18].

These bounds on entanglement have direct implications for quantum information processing. Even if time-dependent control is allowed [19,20], the Lieb-Robinson bound limits the speed at which important states for quantum information and metrology can be prepared [21–25]. In this Letter, we consider the task of using long-range interactions to speed up quantum state transfer, Greenberger-Horne-Zeilinger (GHZ) state preparation, and multiscale entanglement renormalization ansatz (MERA) construction.

State transfer is a process by which an unknown quantum state on one site in a lattice is transferred to another site which is limited by the Lieb-Robinson bound [26–29]. A discussion of possible experimental realizations can be found in Refs. [30–32], and in Ref. [33] a case with long-range interactions is considered. In this Letter, we propose a state transfer protocol which makes use of long-range interactions to transfer a state a distance L on a d-dimensional lattice in a time proportional to L^0 ($\alpha < d$), $\log L$ ($\alpha = d$), $L^{\alpha-d}$ ($d < \alpha \leq d + 1$), or L ($\alpha \geq d$). As an intermediate step of the protocol presented, a GHZ-like state is created, a process also limited by the Lieb-Robinson bound [19]. For polar molecules, Rydberg atoms, or other dipole-dipole interactions in three dimensions, the protocol yields an exponential speedup in the rate of entanglement generation.

As we will discuss, one powerful application of a fast state transfer protocol using long-ranged interactions would be the realization of a circuit described by a MERA [34–36]. MERAs are particularly useful ways to represent entangled states [37–39], such as the ground states of the toric or Haah codes, topological insulators, and quantum Hall states [40–43]. Using dipole-dipole interactions in 3D, our protocol constructs the MERA state exponentially faster than using nearest-neighbor interactions.
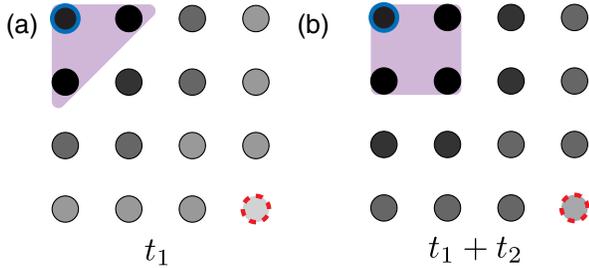
FIG. 1. Our state transfer protocol using long-range inter-actions. We want to move a qubit state from the upper-left site (outlined in solid blue) to the lower-right one (outlined in dashed red). After a time t_1 (a), the nearest-neighbor qubits have shifted from a target to a control (purple region) and continue acting on all other qubits, thereby adding an additional qubit to the set of controls after a further time t_2, as shown in (b). After t_2, each qubit has rotated further (shown by darker shading). The growth continues until the original qubit has effectively performed a CNOT on all qubits in the lattice shown.

State transfer.—Our state transfer protocol first creates a many-body entangled state including the intended starting and final qubits. We do so by applying a controlled X rotation between pairs of qubits (i, j) using a Hamiltonian

$$H_{ij} = h_{ij}(|0\rangle\langle 0|_i \otimes I_j + |1\rangle\langle 1|_i \otimes X_j). \quad (1)$$

Here h_{ij} is the interaction strength, which may not be identical for all pairs of qubits. In Supplemental Material [44], we examine a case where the sign of h_{ij} is variable, but for now we take $h_{ij} > 0$. I_j and X_j are the identity and Pauli X operator acting on qubit j, respectively. When the Hamiltonian in Eq. (1) is applied for a time $t = \pi/(2h_{ij})$, it realizes a controlled-NOT (CNOT) gate between qubits i and j (up to an unimportant phase). In Eq. (1), i is the control qubit for the CNOT, while j is the target qubit. When applied to a control qubit in an arbitrary state and a target qubit in the state $|0\rangle$, the CNOT gate results in a two-qubit state encoding the original qubit:

$$\text{CNOT}(a|0\rangle + b|1\rangle)|0\rangle = a|00\rangle + b|11\rangle. \quad (2)$$

By continuing this process, we can create a many-body entangled state of N qubits $a|0\rangle^{\otimes N} + b|1\rangle^{\otimes N}$ encoding the same state as the initial qubit. The original state can be transferred onto the target qubit by reversing the entangling process and leaving the destination qubit as the final control qubit. If H_{ij} were a nearest-neighbor Hamiltonian, then this procedure would allow us to transfer a qubit state a distance L in $\mathcal{O}(L^\alpha)$ operations, providing a linear scaling which saturates the Lieb-Robinson bound.

By using Hamiltonians with long-range interactions, we can achieve a sublinear state transfer time. We suppose that $h_{ij} = 1/r_{ij}^\alpha$, where r_{ij} is the distance between the qubits i and j [54]. Our protocol (Fig. 1) starts by acting on all qubits in the lattice with a single control qubit storing the initial state. Once the CNOT operation completes on a qubit,

it can be switched from a target to a control and then used to speed up the CNOTs which are still continuing on other qubits. If a single qubit is targeted by many control qubits, then the CNOT operation on that qubit can be completed faster. (Multiple H_{ij} will mutually commute as long as the sets of target qubits and control qubits are disjoint.) If qubit j is targeted by many qubits indexed by i, the time required to complete the CNOT becomes

$$t = \frac{\pi}{2\sum_i h_{ij}} = \frac{\pi}{2\sum_i r_{ij}^{-\alpha}}. \quad (3)$$

(By using dimensionless couplings $h_{ij} = 1/r_{ij}^\alpha$, we are implicitly giving times in units of the inverse nearest-neighbor coupling strength.) In addition to the progressive inclusion of more control qubits, each subsequent qubit has already been rotated by some angle, reducing the remaining time required to complete the operation.

As an example, consider beginning with a system of three qubits arranged in a line:

$$|\psi(t = 0)\rangle = (a|0\rangle + b|1\rangle)|00\rangle. \quad (4)$$

Simultaneously applying H_{12} and H_{13} for a time $t_1 = \pi/2$, the state becomes

$$|\psi(t_1)\rangle = a|000\rangle - ib|11\rangle\left(\cos\frac{\pi}{2^{\alpha+1}}|0\rangle - i\sin\frac{\pi}{2^{\alpha+1}}|1\rangle\right). \quad (5)$$

At this point, the second qubit is made a control, so that the acting Hamiltonians are H_{13} and H_{23}. By continuing the evolution under these Hamiltonians for an additional time

$$t_2 = \frac{\frac{\pi}{2} - \frac{\pi}{2 \cdot 2^\alpha}}{1 + \frac{1}{2^\alpha}} = \frac{\text{rotation remaining}}{\text{sum of interactions}}, \quad (6)$$

the system will end in the final state

$$|\psi(t_1 + t_2)\rangle = a|000\rangle - b|111\rangle. \quad (7)$$

The entire procedure can be reversed, interchanging the roles of qubits 1 and 3, to transfer the original state:

$$|\psi[2(t_1 + t_2)]\rangle = |00\rangle(a|0\rangle + b|1\rangle). \quad (8)$$

We now consider the case of many qubits. First, we specify that we aim to construct a GHZ state across a hypercube whose diagonal spans a distance $L\sqrt{d}$. The points on either end of the diagonal are the original and destination sites for state transfer (see Fig. 2). Because the state transfer time using the protocol in Fig. 1 is difficult to compute, we use a slightly slower protocol that allows us to easily estimate the transfer time both analytically and numerically. Rather than change a qubit into a control as soon as its evolution completes, we instead halt a qubit's evolution when its rotation finishes. Once we have enough qubits to form a full hypercube of controls, we expand the
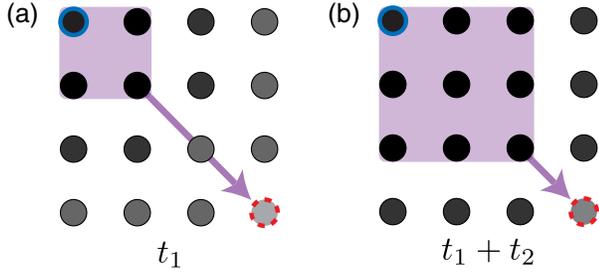
FIG. 2. (a) The suboptimal protocol used for our bounds, with the same color scheme as Fig. 1. After the pth time step, a $(p+1) \times (p+1)$ hypercube of qubits acts as a control. The purple arrow represents $H(2,3)$, as it connects a 2×2 square to a qubit at coordinates $(3,3)$. (b) After time $t_1 + t_2$, another set of qubits has been converted from targets to controls. The purple arrow now represents $H(3,3)$.

control set and continue evolution. This scheme is illustrated in Fig. 2, and we expect it to perform similarly (in terms of the scaling of transfer time) to the scheme in Fig. 1. Let $q = 1, 2, ..., L$ denote each subsequent expansion of the hypercube, so that after time $t = t_1 + t_2 \cdots + t_q$ we can form a complete control hypercube of edge length q. The times t_q are determined by the condition that each qubit must accumulate a total phase of $\pi/2$:

$$\sum_{p=1}^{q} H(p,q) t_p = \frac{\pi}{2}. \tag{9}$$

Here $H(p,q)$ is defined to be the summation of all Hamiltonian strengths h_{ij} for which the control i is in the hypercube with corners $(0,0,0,...)$ and $(p-1, p-1, p-1, ...)$ and the target j is at the site $(q,q,q,...)$ at the corner of a larger hypercube containing the first, as illustrated in Fig. 2. The qubit j is the slowest-evolving qubit on its layer, so its evolution determines the time required to expand the cube in this scheme.

At this point, we will begin looking for bounds on the times t_q. Our first bound arises by noting that for all p, $t_p > t_{p+1}$. This is because, for each p, the quantity $H(p,p)$ is strictly larger than $H(p-1, p-1)$—the qubit at $(p, p, ..., p)$ has more qubits acting on it than its counterpart in the previous step. We use $t_p > t_{p+1}$ to rewrite the phase condition on times in Eq. (9):

$$\frac{\pi}{2} \geq t_q \sum_{p=1}^{q} H(p,q). \tag{10}$$

We now construct two complementary bounds for $H(p,q)$. In some cases (small α), $H(p,q)$ will receive appreciable contributions from the entire hypercube of control qubits. In this case, we can obtain a lower bound by pretending that all control qubits are at the same point a distance $q\sqrt{d}$ away, the maximum possible. However, for large α, the interaction is dominated by nearby qubits, whose contributions are independent of q. For instance, in $H(q,q)$ there is always one qubit at the nearest vertex of the

hypercube whose contribution does not depend on q. These two bounds can be combined to yield

$$H(p,q) \geq \max \left(p^d/(q\sqrt{d})^\alpha, \delta_{pq}/d^{\alpha/2} \right). \tag{11}$$

After substituting Eq. (11) into Eq. (10), the sum can be performed. If we discard all constants depending only on d or α, the result is a bound on the scaling of t_q:

$$t_q \leq \min \left(q^{\alpha - (d+1)}, 1 \right). \tag{12}$$

To obtain the scaling of the entire state transfer process, a sum over t_q is made up to $q = L$. For $\alpha < d$, t_q falls off faster than q^{-1}, so the sum converges to a constant for asymptotic q. The convergence signals that a state can be transferred any desired distance in a constant time. For $\alpha = d$, $t_q = q^{-1}$, so the sum scales logarithmically in L. For $d < \alpha < d+1$, we obtain a polynomial scaling $L^{\alpha - d}$. Finally, for $\alpha \geq d+1$, the constant lower bound on t_q dominates, and the state transfer takes a time proportional to L, just as it does for short-range interacting systems. These scalings are illustrated in Fig. 3 along with the exponents of polynomial fits to the numerical solutions of Eq. (9). Direct use of the long-range interaction between the initial and target qubits would require a time $\mathcal{O}(L^\alpha)$ to transfer a state. Note that our Hamiltonians always obey the condition that $|h_{ij}| \leq r_{ij}^{-\alpha}$, as assumed in previous work on speed limits in long-range interacting systems such as Ref. [14].

Constructing a MERA.—We now demonstrate that our state transfer protocol allows for fast construction of a MERA.

In this context, we will interpret a MERA as a quantum circuit for qubits which acts on successively larger length scales, as shown in Fig. 4, to produce an entangled state from a product state. More general constructions are possible (e.g., with qudits). Our protocol will also apply to a branching MERA [57], provided that after a constant number of layers the circuit disentangles a constant fraction
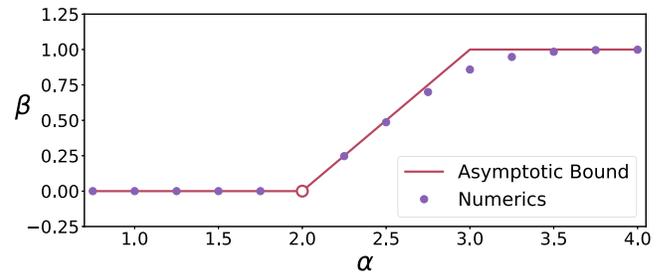

FIG. 3. Numerical results of solving Eq. (9) at different α in $d = 2$. We calculate $\sum_{q \leq L} t_q$ and fit to L^β for L between 900 and 1000; the best-fit exponent is plotted here. The solid line shows the β derived from Eq. (12). At $\alpha = d$ (open circle), the numerics are consistent with the expected logarithmic scaling; the fact that the bound is not saturated at $\alpha = 3$ is due to finite L and should vanish in the $L \to \infty$ limit.
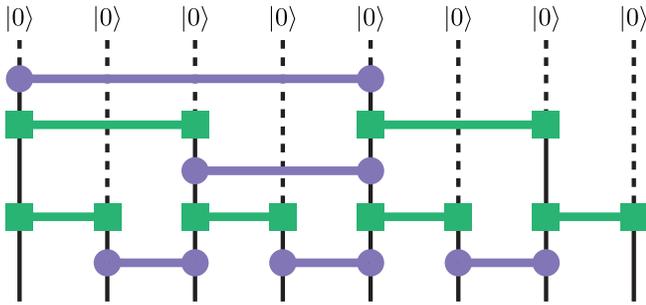
FIG. 4. Sketch of a MERA circuit, with the disentanglers (purple, circle-capped) and isometries (green, square-capped). All qubits begin in the state $|0\rangle$, indicated by a dashed line. At each length scale, entanglement is created or removed to create a many-body entangled state from a product state after $\log_2 L$ steps. Our protocol can be extended to higher dimensions and more complicated tensor structures.

of the remaining qubits to $|0\rangle$, which we need to use to perform our state transfer protocol.

A MERA consists of two alternating types of unitary operations and is easiest to understand in reverse (starting at the bottom of the circuit). The first type of unitary, called a disentangler, removes entanglement at the current length scale. The next operation, an isometry, maps a group of ϕ sites into a single site, leaving the other qubits in the state $|0\rangle$. These operations can be repeated, except that now all unitary gates need to be performed over a distance ϕ times larger than previously.

MERA produces a circuit with depth $\log_\phi L$, but this apparent logarithmic scaling masks an actual time cost due to the continuously increasing length scale. However, we can replace a long-range two-qubit unitary with a state transfer followed by a short-range unitary. The structure of a MERA circuit guarantees that the $|0\rangle$ states required to perform a state transfer will be present between any two qubits when we need to perform a unitary on them.

Suppose that t_τ is the maximum time required to perform a two-qubit gate across a distance ℓ_τ at the τth step of the MERA circuit. We can perform all the MERA operations at a given step in parallel, so a single layer of the MERA simply requires time $2t_\tau$ for the disentanglers and then isometries. The time to perform the entire MERA circuit will then be bounded (up to a constant factor) by

$$t_{\mathrm{MERA}} \lesssim \sum_{\tau=0}^{S-1} t_\tau. \tag{13}$$

Here $S = \log_\phi L$. Our state transfer procedure allows for $t_\tau = 2t_{\mathrm{transfer}}$. The time required to perform the final two-qubit gate does not affect the scaling and so is omitted. We can then bound t_{transfer} by considering the length scale at each step, $\ell_\tau = \phi^\tau$. If $\alpha = d$, t_{transfer} scales as $\log_\phi \ell_\tau$ (as in our state transfer bound but with a constant multiple changing the base of the logarithm), and t_{MERA} will be bounded by $\sim(\log_\phi L)^2$ by considering the largest term in Eq. (13) multiplied by the number of terms. For $\alpha \neq d$,

t_{transfer} scales polynomially in ℓ_τ with exponent β, $t_{\mathrm{MERA}} \lesssim \sum \ell_\tau^\beta$. For $\alpha < d$, $\beta = 0$ and the sum is proportional to $\log_\phi L$. For $\alpha > d$, $\beta = \max(\alpha - d, 1)$. We use $\ell_\tau = \phi^\tau$ and carry out the geometric sum to obtain $t_{\mathrm{MERA}} \lesssim L^\beta$. Thus, we have

α	$\alpha < d$	$\alpha = d$	$d < \alpha \leq d+1$	$\alpha > d+1$
t_{MERA}	$\log_\phi L$	$\log_\phi^2 L$	$L^{\alpha-d}$	L

$$\tag{14}$$

Outlook.—We have demonstrated fast state transfer and MERA construction protocols using long-range interactions. Our protocol's exponential speedup for $\alpha = d$ nearly saturates the bound in Ref. [12], which gives a logarithmic light cone for $\alpha > d$. However, we have not shown that our method is the fastest state transfer protocol possible. Such a result would require demonstrating a general Lieb-Robinson-type bound which we would then saturate. Instead, our protocol limits future Lieb-Robinson bounds for long-range systems. The state transfer protocol we have presented establishes that no finite causal region is possible for $\alpha < d$, since a constant amount of time suffices to establish any desired correlation at arbitrary distances. In previous work, causal regions were seen in systems with $d/2 < \alpha \leq d$ as long as the initial state was not entangled [15]. Like our work, Ref. [15] also uses multiple qubits with long-range interactions to reduce the state transfer time. We have shown that such causal regions do not persist, in general, although it is possible that this violation requires the use of time-dependent Hamiltonians as opposed to the time-independent Hamiltonians in Ref. [15].

For the intermediate value $d < \alpha < d+1$, our protocol shows that no linear light cone can be drawn, although a polynomial bound may be possible. These results should be compared to Ref. [14], which established a polynomial light cone only for $\alpha > 2D$ that becomes linear only in the limit of $\alpha \to \infty$. Our protocol's linear scaling when $\alpha \geq d+1$ suggests that the tightest possible Lieb-Robinson bound may also possess a critical α with a similar property. Resolving this question could reveal important facts about the nature of correlations in long-range interacting systems.

An experimental realization of our protocol could offer significant technological advantages in, for instance, entanglement-enhanced metrology. In Supplemental Material [44], we show how dipole-dipole interactions in three dimensions can be used to implement a variant of our protocol with a focus on Rydberg atoms. Using this protocol, qubits can be entangled exponentially faster than using short-range interactions. In the future, we hope to reduce the local control required to achieve sublinear scaling.

P H Y S I C A L R E V I E W L E T T E R S

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