Rapid Communications

Out-of-time-order correlators in finite open systems

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We study out-of-time-order correlators (OTOCs) of the form $\langle\hat{A}(t)\hat{B}(0)\hat{C}(t)\hat{D}(0)\rangle$ for a quantum system weakly coupled to a dissipative environment. Such an open system may serve as a model of, e.g., a small region in a disordered interacting medium coupled to the rest of this medium considered as an environment. We demonstrate that for a system with discrete energy levels the OTOC saturates exponentially $\propto\sum a_i e^{-t/\tau_i}+\text{const}$ to a constant value at $t\to\infty$, in contrast with quantum-chaotic systems which exhibit exponential growth of OTOCs. Focusing on the case of a two-level system, we calculate microscopically the decay times τ_i and the value of the saturation constant. Because some OTOCs are immune to dephasing processes and some are not, such correlators may decay on two sets of parametrically different time scales related to inelastic transitions between the system levels and to pure dephasing processes, respectively. In the case of a classical environment, the evolution of the OTOC can be mapped onto the evolution of the density matrix of two systems coupled to the same dissipative environment.

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Quantum information spreading in a quantum system is often described by out-of-time-order correlators (OTOCs) of the form

$$K(t)=\langle\hat{A}(t)\hat{B}(0)\hat{C}(t)\hat{D}(0)\rangle, \tag{1}$$

where $\hat{A}$, $\hat{B}$, $\hat{C}$, and $\hat{D}$ are Hermitian operators and $\langle\cdots\rangle$ is the average with respect to the initial state of the system. Correlators of such forms have been first introduced by Larkin and Ovchinnikov [1] in the context of disordered conductors where the correlator $\langle[p_z(t),p_z(0)]^2\rangle$ of particle momenta p_z has been demonstrated to grow exponentially $\propto e^{2\lambda t}$ on times $\tau_0\ll t\ll t_E$ where the exponent λ characterizes the rate of divergence of two classical electron trajectories with slightly different initial conditions, τ_0 is the elastic scattering time, and t_E is the Ehrenfest time, characterizing the crossover between classical and quantum dynamics [2].

The concept of OTOC has revived [3] recently in the context of quantum information scrambling and black holes, motivating further studies of such quantities (see, e.g., Refs. [4–8]). Despite not being measurable observables [9], OTOCs (1) characterize the spreading of quantum information and the sensitivity of the system to the change in the initial conditions. At present, the exponential growth of commutators of the form $\langle[\hat{W}(t),\hat{V}(0)]^2\rangle$ in a sufficiently long interval of time is often used as a definition of quantum chaos (for a discussion of alternative definitions see, e.g., Ref. [10]), and the Lyapunov exponent λ serves as a measure of quantum chaotic behavior in a system. It is also expected that OTOCs may be used [5,11–14] to identify the many-body-localization transition [15].

So far the studies of quantum chaos and information scrambling have been focusing on closed quantum systems. In reality, however, each system is coupled to a noisy environment, which leads to decoherence and affects information spreading. Moreover, a sufficiently strongly disordered interacting system may be separated into a small subsystem of the size of the single-particle localization length or a region of quasilocalized states coupled to the rest of the system considered as the environment. In this Rapid Communication we analyze out-of-time-order correlators in a quantum system weakly coupled to a dissipative environment.

Summary of the results. We demonstrate that, for a system with discrete nondegenerate levels E_n, correlator (1) at long times t exponentially saturates to a constant value of $K(t)\propto\sum a_n e^{i\omega_n t}e^{-t/\tau_n}+\text{const}$, in contrast with chaotic systems where OTOCs contain exponentially growing contributions. The times τ_n describe the rates of information scrambling in the system and are distinct from the dephasing and decoherence times which describe the decay rates of the density matrix. We calculate microscopically the value of the saturation constant and the relaxation times τ_n as a function of the environment's spectral function and the matrix elements of the system-environment coupling. Depending on the choice of operators $\hat{A}$, $\hat{B}$, $\hat{C}$, and $\hat{D}$, the saturation value may be finite or zero. OTOCs relax due to both inelastic transitions between the system's levels and pure dephasing processes caused by slow fluctuations of the energies E_n. Although some OTOCs are immune to dephasing processes, a generic correlator has components both sensitive and insensitive to dephasing and thus decays on two sets of parametrically different scales related to dephasing and relaxation, respectively, as shown in Fig. 1.

Our results suggest, in particular, that a disordered system of interacting particles cannot exhibit quantum chaotic behavior if the typical single-particle level splitting δ_ξ in a volume of linear size ξ (single-particle localization length) exceeds the dephasing rate and the rate of inelastic transitions due to interactions and/or phonons. Thus, chaotic behavior in

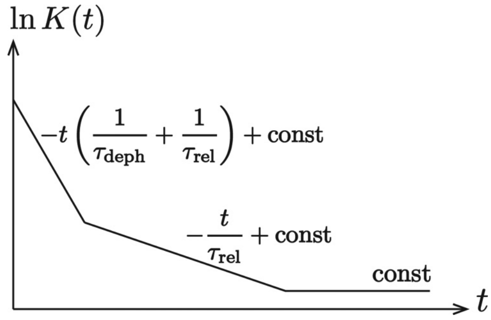

FIG. 1. Time dependence of the out-of-time-order correlator (1) in a system with a significantly larger dephasing rate than relaxation. $\tau_{\rm deph}$ and $\tau_{\rm rel}$ are the (longest) characteristic times of dephasing and inelastic relaxation.

a disordered interacting system requires a sufficiently long single-particle localization length, strong interactions, or, e.g., a phonon bath, which would lead to the quasiparticle decay rate exceeding the level spacing δ_ξ.

For a classical environment, the evolution of an OTOC (1) in an open system may be mapped onto the evolution of the density matrix of two systems coupled to the same environment, which allows one to measure OTOCs by observing the correlations between two systems in a noisy environment, such as spins in a random time-dependent magnetic field.

Model. We consider a system with discrete *nondegenerate* energy levels E_n coupled to a dissipative environment and described by the Hamiltonian,

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{V}\hat{X} + \hat{\mathcal{H}}_{\rm bath}(\hat{X}), \tag{2}$$

where $\hat{\mathcal{H}}_0 = \sum_n E_n |n\rangle\langle n|$ is the Hamiltonian of the system, $\hat{\mathcal{H}}_{\rm bath}(\hat{X})$—the Hamiltonian of the environment, and $\hat{V}\hat{X}$ is the coupling between the system and the environment where the operator $\hat{V} = \sum_{n,m} V_{nm}|n\rangle\langle m|$ acts on the system degrees of freedom and $\hat{X}$ is an environmental variable which commutes with the system degrees of freedom.

To compute the OTOC (1) where operators $\hat{A}$, $\hat{B}$, $\hat{C}$, and $\hat{D}$ act on the system variables, it is convenient to decompose it as $K = K_{m_1m_2,n_1n_2}A_{n_1m_1}C_{n_2m_2}$ (summation over repeated indices implied), where $A_{n_1m_1}$ and $C_{n_2m_2}$ are the matrix elements of operators $\hat{A}$ and $\hat{C}$ and

$$K_{m_1m_2,n_1n_2} = \langle |n_1\rangle\langle m_1|(t)\hat{B}(0)|n_2\rangle\langle m_2|(t)\hat{D}(0)\rangle, \tag{3}$$

where $\langle\cdots\rangle$ is the averaging with respect to both the system and the environment's states.

In the limit of a vanishing system-environment coupling $\hat{V}$, the correlators (3) oscillate with time $K_{m_1m_2,n_1n_2} \propto e^{i(E_{n_1}+E_{n_2}-E_{m_1}-E_{m_2})t}$. A finite coupling between the system and the environment leads to dissipation and relaxation processes and thus to the decay of the elements $K_{m_1m_2,n_1n_2}$. For a weak coupling considered in this Rapid Communication, the characteristic decay times of the OTOCs significantly exceed the correlation time of the environment's degrees of freedom, i.e., of the function $S(t-t') = \langle\hat{X}(t)\hat{X}(t')\rangle_{\rm env}$, and the evolution of the elements is described by a system of Markovian Bloch-Redfield [16] equations of the form [17]

$$\partial_t K_{m_1m_2,n_1n_2} = i(E_{n_1}+E_{n_2}-E_{m_1}-E_{m_2})K_{m_1m_2,n_1n_2} - \sum_{m_1',m_2',n_1',n_2'} \Gamma^{m_1'm_2',n_1'n_2'}_{m_1m_2,n_1n_2} K_{m_1'm_2',n_1'n_2'}. \tag{4}$$

From the definition of the elements (3) it follows that

$$\sum_{m,n} K_{nm,nm} = \langle\hat{B}(0)\hat{D}(0)\rangle = {\rm const}. \tag{5}$$

Equation (5) may also be derived from the microscopic equations of evolution as we show in Ref. [17].

Due to the smallness of the decay rates $\Gamma^{m_1'm_2',n_1'n_2'}_{m_1m_2,n_1n_2}$ in Eq. (4), the evolution of each element $K_{m_1m_2,n_1n_2}$ is affected only by the elements $K_{m_1'm_2',n_1'n_2'}$ with the same oscillation frequency $E_{n_1}+E_{n_2}-E_{m_1}-E_{m_2}$ (secular approximation). In this Rapid Communication we consider systems with sufficiently nondegenerate energy spectra; if two elements oscillate with the same frequency, they may be different only by permutations of indices n_1 and n_2 and/or m_1 and m_2.

For a generic N-level system there are $2N^2-N$ elements (3) with zero energy gaps $E_{n_1}+E_{n_2}-E_{m_1}-E_{m_2}$ (with $m_1=n_1$, $m_2=n_2$ and/or $m_1=n_2$, $m_2=n_1$). These elements are immune to dephasing, i.e., to the accumulation of random phases caused by slow fluctuations of energies E_{n_i}. Such vanishing of dephasing is similar to that in decoherence-free subspaces [18,19] of multiple-qubit systems. We emphasize, however, that even dephasing-immune correlators in general decay at long times due to the environment-induced inelastic transitions between the levels (relaxation processes).

A generic OTOC (1) includes components both sensitive and insensitive to dephasing as well as a component independent of time, which exists due to the conservation law (5). For an environment with a smooth spectral function on the scale of the characteristic level splitting, the characteristic decay rate of the dephasing-immune components may be estimated as $1/\tau_{\rm rel} \sim V_\perp^2 S(\Delta E)$, where $V_\perp$ is the typical off-diagonal matrix element of the perturbation $\hat{V}$ and ΔE is the characteristic level spacing. The other components decay with the characteristic rate of $1/\tau_{\rm deph} + 1/\tau_{\rm rel}$, where $1/\tau_{\rm deph} \sim V_\parallel^2 S(0)$ is the characteristic dephasing rate, where $V_\parallel$ is the typical diagonal matrix element of the perturbation $\hat{V}$. As a result, the decay of the OTOC consist of three stages, corresponding to these characteristic times as illustrated in Fig. 1.

Two-level system. In order to illustrate the meaning of these time scales and the related phenomena, we focus below on the case of a two-level system, equivalent to a spin 1/2 in a random magnetic field (we provide microscopic analysis of OTOCs for a generic multilevel system in Ref. [17]), described by the Hamiltonian,

$$\hat{\mathcal{H}} = \frac{1}{2}B\hat{\sigma}_z + \frac{1}{2}\hat{\boldsymbol{\sigma}}\mathbf{n}\hat{X} + \hat{\mathcal{H}}_{\rm bath}(\hat{X}), \tag{6}$$

where $\hat{\boldsymbol{\sigma}}$ is a vector of Pauli matrices and $\mathbf{n}$ is a constant unit vector, the direction of the fluctuations of the magnetic field.

The dissipative environment induces transitions $|\uparrow\rangle \to |\downarrow\rangle$ with the rate $\Gamma_\downarrow = \frac{1}{4}(n_x^2+n_y^2)S(B)$ as well as the opposite transitions $|\downarrow\rangle \to |\uparrow\rangle$ with the rate $\Gamma_\uparrow = \frac{1}{4}(n_x^2+n_y^2)S(-B)$, where $S(\omega)$ is the environment spectrum, the Fourier transform of $S(t-t') = \langle\hat{X}(t)\hat{X}(t')\rangle_{\rm env}$. Weak fluctuations of the magnetic field in the longitudinal direction lead to dephasing with the rate $\Gamma^{\phi} = \frac{1}{2} n_z^2 S(0)$. We focus below on the long-time dynamics of the system and assume for simplicity that the rate Γ^{ϕ} of pure dephasing significantly exceeds the rates $\Gamma_{\uparrow}$ and $\Gamma_{\downarrow}$ of inelastic transitions between the levels of the spin; in the opposite case, all OTOC decay rates are on the same order of magnitude.

The OTOCs $K_{\uparrow\uparrow,\downarrow\downarrow}$ and $K_{\downarrow\downarrow,\uparrow\uparrow}$ oscillate with frequencies $\pm 2(E_{\downarrow} - E_{\uparrow}) = \mp 2B$ and have dephasing rate $4\Gamma^{\phi}$, the same as ± 1-projection states of a spin 1 in magnetic-field B,

$$K_{\uparrow\uparrow,\downarrow\downarrow}, K_{\downarrow\downarrow,\uparrow\uparrow} \propto e^{\mp 2iBt} e^{-4\Gamma^{\phi} t}, \tag{7}$$

where we have neglected the small relaxation rates $\Gamma_{\uparrow,\downarrow} \ll \Gamma^{\phi}$.

There are eight elements (3) which correspond to three spin indices pointing in one direction and one spin index pointing in the opposite direction. These elements oscillate with frequencies $\pm B$ and have the same dephasing rate as a spin $1/2$,

$$K_{\uparrow\downarrow,\downarrow\downarrow}, K_{\downarrow\uparrow,\uparrow\uparrow}, K_{\downarrow\downarrow,\uparrow\downarrow}, \ldots, \propto e^{-\Gamma^{\phi} t}. \tag{8}$$

The behavior of OTOCs at long-times $t \gg 1/\Gamma^{\phi}$ is determined by the components with a vanishing frequency $E_{n_1} + E_{n_2} - E_{m_1} - E_{m_2}$ of coherent oscillations because such components are insensitive to dephasing. For a spin $1/2$, their evolution is described by the system of equations (as follows from the generic master equations [17]):

$$\partial_t \begin{pmatrix} K_{\downarrow\uparrow,\uparrow\downarrow} \\ K_{\uparrow\downarrow,\downarrow\uparrow} \\ K_{\uparrow\downarrow,\uparrow\downarrow} \\ K_{\downarrow\uparrow,\downarrow\uparrow} \\ K_{\uparrow\uparrow,\uparrow\uparrow} \\ K_{\downarrow\downarrow,\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} -\Gamma_{\downarrow} - \Gamma_{\uparrow} & 0 & -\Gamma_{\downarrow} & -\Gamma_{\downarrow} & \Gamma_{\downarrow} & \Gamma_{\downarrow} \\ 0 & -\Gamma_{\downarrow} - \Gamma_{\uparrow} & -\Gamma_{\uparrow} & -\Gamma_{\uparrow} & \Gamma_{\uparrow} & \Gamma_{\uparrow} \\ -\Gamma_{\uparrow} & -\Gamma_{\downarrow} & -\Gamma_{\downarrow} - \Gamma_{\uparrow} & 0 & \Gamma_{\downarrow} & \Gamma_{\uparrow} \\ -\Gamma_{\uparrow} & -\Gamma_{\downarrow} & 0 & -\Gamma_{\downarrow} - \Gamma_{\uparrow} & \Gamma_{\downarrow} & \Gamma_{\uparrow} \\ \Gamma_{\uparrow} & \Gamma_{\downarrow} & \Gamma_{\uparrow} & \Gamma_{\uparrow} & -2\Gamma_{\downarrow} & 0 \\ \Gamma_{\uparrow} & \Gamma_{\downarrow} & \Gamma_{\downarrow} & \Gamma_{\downarrow} & 0 & -2\Gamma_{\uparrow} \end{pmatrix} \begin{pmatrix} K_{\downarrow\uparrow,\uparrow\downarrow} \\ K_{\uparrow\downarrow,\downarrow\uparrow} \\ K_{\uparrow\downarrow,\uparrow\downarrow} \\ K_{\downarrow\uparrow,\downarrow\uparrow} \\ K_{\uparrow\uparrow,\uparrow\uparrow} \\ K_{\downarrow\downarrow,\downarrow\downarrow} \end{pmatrix}. \tag{9}$$

The rates of the long-time decay of OTOCs are given by the eigenvalues of the matrix in Eq. (9) (with the minus sign) and are shown (except for the zero eigenvalue) in Fig. 2. Such a matrix always has a zero eigenvalue due to the conservation law (5). The system also has a triply degenerate decay rate of $\Gamma_{\uparrow} + \Gamma_{\downarrow}$. The other two decay rates are given by $\frac{1}{2}[3\Gamma_{\uparrow} + 3\Gamma_{\downarrow} \pm (\Gamma_{\uparrow}^2 + 34\Gamma_{\uparrow}\Gamma_{\downarrow} + \Gamma_{\downarrow}^2)^{1/2}]$.

At long-times $t \to \infty$ the correlator (1) saturates to a constant value determined by the projection of the OTOC (1) on the zero-decay-rate mode,

$$K(t \to \infty) = \frac{1}{\sqrt{2\Gamma_{\uparrow}^2 + 2\Gamma_{\downarrow}^2}} (\Gamma_{\downarrow} A_{\uparrow\downarrow} C_{\downarrow\uparrow} + \Gamma_{\uparrow} A_{\downarrow\uparrow} C_{\uparrow\downarrow} + \Gamma_{\uparrow} A_{\uparrow\uparrow} C_{\uparrow\uparrow} + \Gamma_{\downarrow} A_{\downarrow\downarrow} C_{\downarrow\downarrow}). \tag{10}$$

Although we assumed a small inelastic relaxation rate in comparison with the dephasing rate, we emphasize that the result (10) for the saturation value of the OTOC holds for an arbitrary ratio of dephasing and relaxation rates.

Mapping to the evolution of two systems for a classical environment. The evolution of the OTOCs (3) is similar to that of the density-matrix elements,

$$\rho_{m_1 m_2, n_1 n_2} = \langle \langle |m_1\rangle \langle n_1| (t) \rangle_{\mathrm{Sys}_1} \langle |m_2\rangle \langle n_2| (t) \rangle_{\mathrm{Sys}_2} \rangle_X, \tag{11}$$

of a compound system consisting of two identical subsystems ("Sys_1" and "Sys_2") coupled to the same dissipative environment, where m_1 and n_1 and m_2 and n_2 in Eq. (11) are the states of the first and the second subsystems, respectively, $|m_i\rangle \langle n_i| (t)$ is an operator in the interaction representation, and $\langle \cdots \rangle_X$ is the averaging with respect to the environment's degrees of freedom. The Hamiltonian of such a compound system is given by

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 \otimes \mathbb{1} + \mathbb{1} \otimes \hat{\mathcal{H}}_0 + (\hat{V} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{V}) \hat{X} + \hat{\mathcal{H}}_{\mathrm{bath}}(\hat{X}), \tag{12}$$

where $\cdots \otimes \cdots$ is the product of the subsystem subspaces; $\hat{\mathcal{H}}_0$ and $\hat{V}$ are the Hamiltonian of each subsystem and its coupling to the environment, and the environment's variable $\hat{X}$ commutes with all degrees of freedom of subsystems Sys_1 and Sys_2.

The evolution of the elements (3) and (11) is described by similar Markovian master equations (see Ref. [17] for a microscopic derivation). In particular, in the limit of a classical environment $[\langle \hat{X}(t) \hat{X}(t') \rangle_{\mathrm{env}} = \langle \hat{X}(t') \hat{X}(t) \rangle_{\mathrm{env}}]$, the evolution of OTOCs (3) can be mapped exactly onto that of the density-matrix (11) of two systems coupled to this environment as follows from the definitions of these quantities. The conservation law (5) is mapped then onto the conservation of the trace of the density matrix of a compound system consisting of two subsystems.

In the limit of a classical environment, the spectral function is even $S(\omega) = S(-\omega)$, and the relaxation rate $i \to j$ for each pair of levels i and j in a system matches the reverse rate $j \to i$. In particular, in the case of a two-level system,

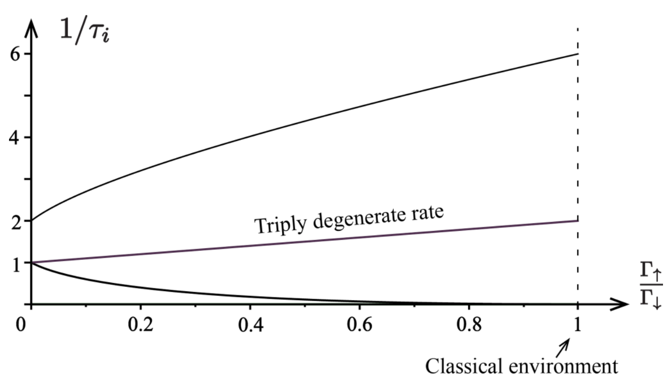

FIG. 2. Nonzero rates (in units of $\Gamma_{\downarrow}$) of long-time decay of out-of-time-order correlators in a two-level system as a function of the ratio $\Gamma_{\uparrow}/\Gamma_{\downarrow}$ of the transition rates between the system levels.

$\Gamma_\uparrow = \Gamma_\downarrow = \Gamma$, and the OTOC has three decay rates at long-times $t \gg \Gamma^{\phi^{-1}}$: $\Gamma_1 = 6\Gamma$, $\Gamma_2 = 2\Gamma$ (triply degenerate), and $\Gamma_3 = 0$ (doubly degenerate), as shown in Fig. 2. Due to the mapping, these rates match the decay rates of pairwise correlators of observables in, e.g., an ensemble of spins in a uniform random magnetic field and thus may be conveniently measured in such ensembles.

We emphasize that the mapping between an OTOC and the evolution of two subsystems coupled to the same classical environment holds for an arbitrary system-environment coupling but not only in the limit of the weak coupling considered in this Rapid Communication. This mapping suggests a way for measuring OTOCs in generic systems in the presence of classical environments through observing correlators $\langle\langle\hat{A}(t)\rangle_{\text{Sys}_1}\langle\hat{C}(t)\rangle_{\text{Sys}_2}\rangle_X$ of observables $\hat{A}$ and $\hat{C}$ between two systems.

Discussion. We computed OTOCs in a finite system weakly coupled to a dissipative environment. The model of such a finite open system may be used to understand OTOC behavior in sufficiently strongly disordered interacting media (in the presence or in the absence of a phonon bath).

A material with weak short-range interactions and strong disorder, which leads to the localization of all single-particle states, exhibits insulating behavior at low temperatures [15]. Local physical observables in such a system are strongly correlated only on short length scales, and their properties may be understood by considering a single "localizaton cell," particle states in a region of space of the size of order of the (single-particle) localization length ξ, which may be assumed weakly coupled to the rest of the system.

The energy spectrum of the localization cell may be probed via response functions of local operators in the cell, e.g., the response function $\chi(\omega) = \sum_{\alpha,\beta} \frac{(f_\alpha - f_\beta)|Q_{\alpha\beta}|^2}{E_\alpha - E_\beta + \omega + i0}$ of the charge Q in a region inside the cell to the voltage in this region, where E_α and E_β are the energies of many-body states and f_α is their distribution function. For temperatures lower than a critical value, quasiparticles in the system have a zero-decay rate [20] ("superinsulating" regime [15]), and the system thus responds only at a discrete set of frequencies $\omega = E_i - E_j$, determined by the energy gaps between many-body states as shown in Fig. 3. The OTOC (1) in this regime oscillates $K(t) \propto \sum_n a_n e^{i\omega_n t}$ with a discrete set of frequencies $\omega_n = E_{i_n} + E_{i'_n} - E_{j_n} - E_{j'_n}$. In the limit of a very large number of levels in a localization cell, the sum of such oscillating contributions approaches a smooth function with exponentially growing contributions describing the single-particle chaos of Ref. [1], so long as dynamics on time scales shorter than the Thouless scale $t_{\text{Th}} = \xi^2/D$ is neglected. However, the energy levels remain discrete in the superinsulating regime.

When the temperature (or the interaction strength at a given temperature) exceeds a critical value, the levels and response functions get broadened ("metallic" phase [15]) as illustrated in Fig. 3, becoming smoother with increasing temperature and/or interactions. Near the superinsulator-metal transition the characteristic level width Γ is significantly smaller than the gaps between levels, and the localization cell may be considered as an open system weakly coupled to a dissipative environment. The same model may be applied also to a strongly disordered material with an external bath, such as a system of phonons, which provide a finite level width Γ at all finite temperatures. The local operators $\hat{A}$, $\hat{B}$, $\hat{C}$, and $\hat{D}$ in Eq. (1) do not necessarily act on states in one localization cell but may involve states in several cells close to each other. These cells may still be considered as a single quantum dot in a noisy environment so long as the level spacing in the dot exceeds the level width. Such a model of an open quantum dot may be also realized directly, e.g., using superconducting qubits or trapped cold atoms.

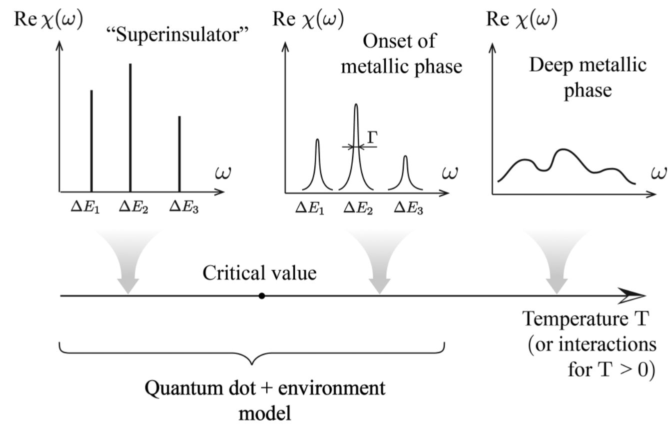

FIG. 3. Response of a localization cell in an interacting system for various temperatures T (or interaction strengths for $T > 0$). ΔE_i's are the energy gaps between the many-body levels in the cell, and Γ is the level width.

The results of this Rapid Communication suggest, in particular, that a strongly disordered system with the level spacing δ_ξ exceeding the effective dissipation rates Γ is nonchaotic. Whereas our results apply to weakly conducting and insulating materials for which the system-environment coupling may be considered small, we leave it for a future study whether nonchaotic behavior persists in systems strongly coupled to the environment (corresponding to an effectively continuous energy spectrum of a localization cell).

For a classical environment, the evolution of an OTOC in an open system matches the evolution of the correlators of observables between two identical systems coupled to the same environment, which may be used for measuring OTOCs in open systems in classical environments. The possibility to develop a similar measurement method for the case of a quantum environment is another question which deserves further investigation.

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