Kitaev Chains with Long-Range Pairing

Davide Vodola,[1,2] Luca Lepori,[1] Elisa Ercolessi,[2] Alexey V. Gorshkov,[3] and Guido Pupillo[1]
[1]*IPCMS (UMR 7504) and ISIS (UMR 7006), Université de Strasbourg and CNRS, 67000 Strasbourg, France*
[2]*Dipartimento di Fisica, Università di Bologna and INFN, Via Irnerio 46, 40126 Bologna, Italy*
[3]*Joint Quantum Institute, NIST/University of Maryland, College Park, Maryland 20742, USA*
(Received 21 May 2014; published 9 October 2014)

We propose and analyze a generalization of the Kitaev chain for fermions with long-range p-wave pairing, which decays with distance as a power law with exponent α. Using the integrability of the model, we demonstrate the existence of two types of gapped regimes, where correlation functions decay exponentially at short range and algebraically at long range ($\alpha > 1$) or purely algebraically ($\alpha < 1$). Most interestingly, along the critical lines, long-range pairing is found to break conformal symmetry for sufficiently small α. This is accompanied by a violation of the area law for the entanglement entropy in large parts of the phase diagram in the presence of a gap and can be detected via the dynamics of entanglement following a quench. Some of these features may be relevant for current experiments with cold atomic ions.

DOI: 10.1103/PhysRevLett.113.156402 PACS numbers: 71.10.Pm, 03.65.Ud, 67.85.−d, 85.25.−j

The Kitaev chain describes the dynamics of one-dimensional spinless fermions with superconducting p-wave pairing [1]. Open Kitaev chains support unpaired Majorana modes exponentially localized at each end [2], implying the existence of a topological superconducting phase [3]. Their probable recent observation in spin-orbit coupled semiconductors [4,5] has sparked renewed interest in novel properties of topological models, as well as in experimental realizations. For example, Kitaev chains with long-range (LR) hopping and pairing have recently been proposed as models for helical Shiba chains, made of magnetic impurities on an s-wave superconductor [6].

Intimately related to the Kitaev chain, Ising-type spin chains with tunable LR interactions can now be realized using trapped ions coupled to motional degrees of freedom or, alternatively, using neutral atoms coupled to photonic modes [7–11]. Very recently, theory and experiments have provided evidence for novel static and dynamic phenomena in these systems, such as, e.g., the nonlocal propagation of correlations [8,9,12–14] or the possible violation of the area law in one dimension [15]. While some of these phenomena can be explained theoretically using approximate analytical and numerical methods [15,16], it remains a fundamental challenge to determine basic properties of LR interacting systems, where methods based on short-range (SR) models may fail.

In this Letter, we introduce and analyze an exactly solvable model for one-dimensional fermions with LR pairing, decaying with distance r as $\sim 1/r^{\alpha}$. We analyze the phase diagram as a function of the power α of the pairing, finding several novel features: (i) gapped phases for $\alpha > 1$ where the decay of correlation functions evolves from exponential to algebraic from short to long distances and (ii) a gapped phase with a purely algebraic decay of correlations for $\alpha < 1$. For the open chain, we find that (iii) the localization of the edge modes, similar to the case of correlations, varies from hybrid (exponential followed by algebraic) for $\alpha > 1$ to purely algebraic for $\alpha < 1$, where these modes become gapped. Throughout the phase diagram, (iv) the entanglement entropy fails to capture some of the main features of the energy spectrum and correlation functions. However, it correctly predicts (v) an exotic transition along one of the two critical lines induced by LR pairing from an Ising-type theory for $\alpha > 3/2$ to a Luttinger-liquid-type theory for $\alpha < 3/2$. This corresponds to (vi) a breaking of conformal symmetry, which can also be inferred by looking at the entanglement dynamics after a quench. Finally, we discuss the relevance of these results to Ising-type chains studied in trapped-ion experiments [8,9].

We consider the following fermionic Hamiltonian on a lattice of length L (with unit lattice constant):

$$H_L = -t\sum_{j=1}^{L}(a_j^{\dagger}a_{j+1} + \mathrm{H.c.}) - \mu\sum_{j=1}^{L}\left(n_j - \frac{1}{2}\right) + \frac{\Delta}{2}\sum_{j=1}^{L}\sum_{\ell=1}^{L-1} d_{\ell}^{-\alpha}(a_j a_{j+\ell} + a_{j+\ell}^{\dagger}a_j^{\dagger}). \tag{1}$$

$a_j^{\dagger}$ (a_j) is a fermionic creation (annihilation) operator on site j, $n_j = a_j^{\dagger}a_j$, t the tunneling rate, μ the chemical potential, and Δ the strength of the p-wave pairing. For a closed chain, we define $d_{\ell} = \ell$ ($d_{\ell} = L - \ell$) if $\ell < L/2$ ($\ell > L/2$) and choose antiperiodic boundary conditions [17]. For an open chain, $d_{\ell} = \ell$ and we drop terms containing $a_{j>L}$. We set $\Delta = 2t = 1$ [18].

Hamiltonian (1) has a rich phase diagram which, when the pairing is between nearest neighbors only, coincides—via the Jordan-Wigner transformation—with that of the

XY model, a generalization of the SR Ising model [19], belonging to its universality class [20] and sharing with it gapped ferromagnetic ($|\mu| < 1$) and paramagnetic phases ($|\mu| > 1$), separated by two critical points at $\mu = \pm 1$ [21]. Furthermore, the unitary transformation $a_i \to (-1)^i a_i^\dagger$ ensures that the phase diagram is identical for positive and negative μ.

For the LR Ising model, recent numerical results have shown algebraic decay of correlation functions in the gapped paramagnetic phase [15], in agreement with results from other similar spin models [22–26]. For any finite α, however, a Jordan-Wigner transformation does not connect Hamiltonian (1) to the XY model, implying that their respective phase diagrams can be different. Moreover, for finite α, the transformation $a_i \to (-1)^i a_i^\dagger$ no longer connects positive and negative μ, meaning that the phase diagram may no longer be symmetric across the line $\mu = 0$. In the following, we determine the phase diagram of (1) by analyzing its energy spectrum, the entanglement entropy, and the decay of correlation functions for the closed chain, and then the edge modes for the open one.

The spectrum of excitations, obtained via a Bogoliubov transformation, is

$$\lambda_\alpha(k_n) = \sqrt{(\cos k_n + \mu)^2 + f_{k_n,\alpha}^2}. \quad (2)$$

$k_n = 2\pi(n + 1/2)/L$ are the lattice (quasi)momenta with $0 \le n < L$ and the functions $f_{k,\alpha}^L \equiv \sum_{l=1}^{L-1} \sin(kl)/d_\ell^\alpha$. These functions can also be evaluated in the thermodynamic limit [27]. The ground state of (1) is $|\mathrm{GS}\rangle = \prod_{n=0}^{L/2-1} (\cos\theta_{k_n} - i \sin\theta_{k_n} a_{k_n}^\dagger a_{-k_n}^\dagger)|0\rangle$, with $\tan(2\theta_{k_n}) = -f_{k_n,\alpha}/(\cos k_n + \mu)$.

As expected from the SR Kitaev model [1,21], Eq. (2) is gapped for all $\alpha > 1$, except for $|\mu| = 1$. When $\alpha \le 1$, the situation changes, the most evident effect being the fact that along the line $\mu = -1$ the model becomes massive, as one can see from (2) for $k = 0$. As a consequence, by tuning α and μ, it is now possible to continuously connect the paramagnetic and ferromagnetic phases of the SR Kitaev model, without closing the gap. (Without leaving the $\alpha \to \infty$ limit, such a gapped path can only be achieved with two Kitaev wires [28].) In contrast, the $\mu = 1$ critical line also survives for $\alpha \le 1$, but the nature of the phase transition changes drastically, as we argue below.

These features are summarized in the phase diagram of Fig. 1(a). Using the method of [29],[40] we compute the von Neumann entropy $S(L/2) = -\mathrm{tr}(\rho_{L/2} \log \rho_{L/2})$, where $\rho_{L/2}$ is the half-chain reduced density matrix. For one-dimensional SR gapped systems, $S(L/2)$ saturates to a constant value, a behavior known as the area law [30,31] and associated with an exponential decay of correlation functions [32]. On the other hand, in conformally invariant models, $S(L/2)$ scales according to $S(L/2) = (c/3) \log L + b$, with b being a nonuniversal term and c the central charge [33,34]. In particular, $c = 1/2$ for $|\mu| = 1$ for the SR Kitaev chain.

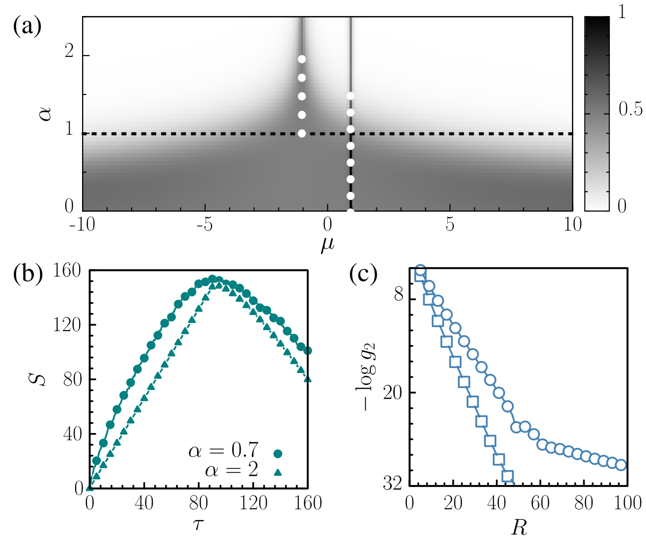

FIG. 1 (color online). (a) Effective central charge c_{eff} obtained by fitting $S(L/2)$. Two gapless conformal field theories with $c = 1/2$ are visible for $\mu = 1$ ($\alpha > 3/2$) and $\mu = -1$ ($\alpha > 2$). White vertical dotted lines: gapless lines with broken conformal symmetry. Horizontal dashed line separates two regions: correlation functions display a hybrid exponential-algebraic ($\alpha > 1$) and purely algebraic decay ($\alpha < 1$). (b) Time evolution of $S(L/2)$ after a quench from a product state with $\mu \gg 1$ to $\mu = 1$: $\alpha > 1$, $S(L/2)$ grows linearly, $\alpha < 1$, $S(L/2)$ grows logarithmically. (c) $g_2(R)$ correlation function for $\mu = 2$ and $\alpha = 10$ (squares), showing exponential behavior and $\alpha = 7$ (circles), showing an exponential with an algebraic tail even in the gapped region.

Performing finite-size scaling (see the Supplemental Material [35]), we find that for all α and μ, $S(L/2)$ is well approximated by $S(L/2) = (c_{\mathrm{eff}}/3) \log L + b$, where c_{eff} is the effective central charge [plotted in Fig. 1(a)]. In particular, (i) for $\alpha > 1$, $c_{\mathrm{eff}} = 0$ almost everywhere in the gapped region $|\mu| \neq 1$. However, logarithmic deviations are important close to the critical line $\mu = -1$ for $\alpha < 2$ [see Fig. 1(a) and below], signaling a violation of the area law. (ii) For $\alpha < 1$, $c_{\mathrm{eff}} \neq 0$ within the gapped region. This effect is particularly evident for $|\mu| \lesssim 1$, while $c_{\mathrm{eff}} = 0$ for $|\mu| \to \infty$.

In addition, most interestingly, (iii) along the critical line $\mu = 1$ we observe an increase of c_{eff}, obtained by $S(L/2)$, from $c_{\mathrm{eff}} = 1/2$ when $\alpha > 3/2$ to $c_{\mathrm{eff}} = 1$ when $\alpha = 0$ (see the Supplemental Material [35]). In a conformal field theory (CFT), the latter would correspond to a Luttinger-liquid-type theory. Indeed, we have verified numerically that $g_2(R)$ [see Fig. 2(b) and below] displays a strongly dimerized behavior in this region, similar to that of a charge-density wave. This peculiar behavior is further corroborated by an exact analytical computation for $\alpha = 0$ [35]. We demonstrate below that this behavior is linked to the breaking of conformal symmetry for $\alpha \le 3/2$.

The violations of the area law despite the presence of a gap could be naively regarded as a failure of S to capture

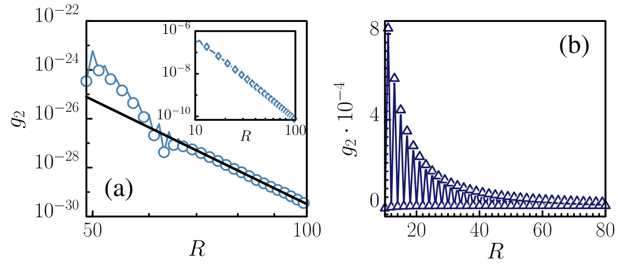

FIG. 2 (color online). (a) Long-distance behavior of $g_2(R)$ for $\alpha = 7$ and $\mu = 2$ in log-log scale, displaying algebraic decay. Continuous line: analytic prediction $\sim 1/R^{14}$. [Figure 1(c) shows the same data set on a log plot.] (Inset) Purely algebraic decay for $\alpha \leq 1$. Here, $\mu = 2$ and $\alpha = 0.5$. (b) $g_2(R)$ for $\mu = 1$ and $\alpha = 0.5$, displaying algebraic decay with oscillating behavior.

the physics of the model at small α. We recall that similar behavior has previously been found for both massive quasifree fermionic models and Ising chains [15]. Thus, from a different perspective, we may argue that S (together with the correlation functions which we will discuss below) can capture a fundamental change in the nature of the ground state, when moving towards very LR interactions.

For small α, LR pairing becomes dominant, and its presence shows up in the physical behavior of nonlocal quantities, such as S and correlation functions, but cannot be inferred simply from the structure of the spectrum [41]. On the line $\mu = 1$, this leads to a breakdown of conformal invariance, even if the spectrum remains linear about the Fermi momentum k_F, as can be seen by looking at finite-size corrections to the ground-state energy density $e(\alpha) = -\sum_{n=0}^{L/2-1} \lambda_\alpha(k_n)/L$. The latter can be computed with the Euler-MacLaurin formula to give [42]

$$e(\alpha) = e_\infty(\alpha) + \pi[\lambda'_\alpha(\pi) - \lambda'_\alpha(0)]/(12L^2), \qquad (3)$$

where $e_\infty(\alpha) = -\frac{1}{\pi}\int_0^{\pi/2} \lambda_\alpha(2x)dx$ is the value of $e(\alpha)$ in the thermodynamic limit. Exact calculations (see the Supplemental Material [35]) show that, for all $\alpha > 3/2$, $\lambda'_\alpha(0) = 0$, and thus one recovers the standard CFT result $e(\alpha) = e_\infty(\alpha) - \pi v_F c/(6L^2)$, where v_F is the Fermi velocity and $c = 1/2$, in agreement with the expected value of c for the SR Ising model [20,43]. For $\alpha = 1$, however, $\lambda'_1(0)$ does not vanish and results in a value $c_{\rm eff}$ different from that computed from $S(L/2) \sim (c_{\rm eff}/3)\log L$, and depends on Δ. This nonuniversal behavior signals a breaking of CFT and is also accompanied by the violation of the area law close to the critical line found in (i) above. Breaking of CFT is most evident for $\alpha < 3/2 (\neq 1)$, where $\lambda'_\alpha(0)$ is found to diverge. A similar behavior arises at $\mu = -1$ for $1 < \alpha < 2$: the scaling (3) fails since the contribution from $\lambda'_\alpha(0)$ diverges (see the Supplemental Material [35]).

Following the ideas of Ref. [13], conformal invariance along $\mu = 1$ for $\alpha < 1$ can also be tested by looking at the time dependence of S after a quench from $\mu \gg 1$ to $\mu = 1$. This is shown in Fig. 1(b), from which it is evident that S grows linearly with time τ if $\alpha > 1$, as predicted by CFT [44], whereas it grows only logarithmically with τ when $\alpha < 1$. We note that a logarithmic growth of S has recently been theoretically demonstrated in Ref. [13] for the LR Ising chain [8,9]. We come back to this point below.

Correlation functions can be used to further clarify the phase diagram. The one-body correlation $g_1(|i-j|) = \langle a_i^\dagger a_j \rangle$ and the anomalous one $g_1^a(|i-j|) = \langle a_i^\dagger a_j^\dagger \rangle$ can be computed semianalytically for finite L, as well as in the thermodynamic limit (see the Supplemental Material [35]). The density-density correlation $g_2(|i-j|) = \langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle = g_1^a(|i-j|)^2 - g_1(|i-j|)^2$ is then immediately obtained from Wick's theorem. Examples of $g_2(R)$ in the regions $\alpha > 1$ and $\alpha < 1$ are shown in Figs. 1(c) and 2(b), respectively. In particular, Fig. 1(c) illustrates the behavior of $g_2(R)$ in the gapped phase with $\mu = 2$ for $\alpha = 7$ and $\alpha = 10$. While at $\alpha = 10$ the behavior seems purely exponential, similar to SR gapped systems, the case $\alpha = 7$ shows that the decay of $g_2(R)$ varies from an initial exponential one to an algebraic one for large R. This hybrid exponential-algebraic decay is consistent with the recent hybrid exponential-algebraic Lieb-Robinson bounds on the propagation of information in systems with power-law interactions [16]. We find numerically that in our system this hybrid exponential-algebraic decay is characteristic of *all* correlation functions for finite $\alpha > 1$, and we obtain for the general asymptotic behavior [see Fig. 2(a)] $g_2(R) \sim 1/R^{2\alpha}$, $g_1(R) \sim 1/R^{\alpha+1}$, and $g_1^a(R) \sim 1/R^\alpha$.

These results are confirmed analytically in the thermodynamic limit, where, for example, $g_1(R)$ reads

$$g_1(R) = -\frac{1}{\pi}\mathrm{Re}\int_0^\pi dk e^{ikR}\mathcal{C}_\alpha(k), \qquad (4)$$

with $\mathcal{C}_\alpha(k) = (\cos k + \mu)/(2\lambda_\alpha(k))$. Integrating by parts, one finds that the leading contribution to Eq. (4) decays as $1/R^{n+1}$, with n being the order of the first nonvanishing odd derivative of $\mathcal{C}_\alpha(k)$ at $k = 0$ [45]. When $\alpha > 1$ is an odd integer, $n = \alpha$. A similar reasoning applies to $g_1^a(R)$, with $n = \alpha - 1$. We finally note that the long-distance behavior of $g_2(R)$ is identical to that of the two-point correlation function of the LR Ising chain, numerically found in [15]. Such a prediction and similar ones can be derived for this model within the spin-wave approximation [46].

The most surprising behavior occurs for $\alpha \leq 1$, where the correlation functions display purely algebraic decay at all length scales, as illustrated for $g_2(R)$ in Fig. 2(b). The fact that the behavior of the system changes when α falls below 1 is further illustrated in Fig. 3(a). We plot the numerically obtained exponent γ of the algebraic decay of $g_1(R) \sim R^{-\gamma}$ as a function of α at fixed μ: a discontinuity occurs at $\alpha = 1$ for all values of μ. Similarly, Fig. 3(b) shows that the scaling exponent δ of $g_2(R) \sim R^{-\delta}$ becomes $\delta = 2$ for every

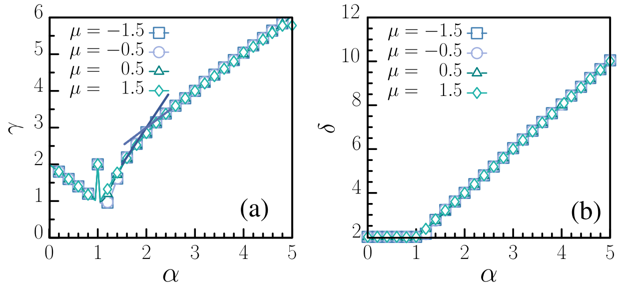

FIG. 3 (color online). Exponents (a) γ and (b) δ of the algebraic decay of $g_1(R)$ and $g_2(R)$ vs α, obtained by fitting with power-law functions, namely, $g_1(R) \sim R^{-\gamma}$ and $g_2(R) \sim R^{-\delta}$. The equations of the two straight lines in (a) are $2\alpha - 1$ and $\alpha + 1$.

$\alpha \leq 1$. Apart from finite-size effects that might be relevant close to $\alpha = 1$, the exponents γ and δ are found to be independent of μ. Notably, the change of behavior at $\alpha = 1$ is not detected properly by S. Finally, for the case $\mu = 1$ and $\alpha \leq 3/2$, integrals as in Eq. (4) receive contributions from both momenta $k = 0$ and $k = \pi$, resulting in the observed dimerized behavior of correlation functions [Fig. 2(b)] (see the Supplemental Material [35]).

We note that, in the thermodynamic limit, we find a divergent velocity of high-energy quasiparticles (at $k = 0$) for $\alpha < 3/2$ and $\mu \neq -1$ and for $\alpha < 2$ at $\mu = -1$ (see the Supplemental Material [35]). While these do not contribute to spectral properties such as the gap, they do affect the behavior of quantities such as correlation functions, entanglement entropy, and postquench evolution [47]. For example, Fig. 3(a) shows that the exponent γ for $g_1(R)$ changes behavior for $\alpha \lesssim 2$ with respect to the predicted value $\gamma = \alpha + 1$. Related effects are also at the origin of the violation of the Lieb-Robinson bound [14,48] recently observed in LR Ising-type models [8,9,12].

Open boundary conditions.—Majorana edge modes, related to the $\mathcal{Z}_2$ symmetry of (1), arise for $|\mu| < 1$ if $\alpha \to \infty$ [1]. At finite α, the Hamiltonian still exhibits this symmetry, and the edge modes are again expected. For $\alpha \gtrsim 1$, the decay of the square of the edge-mode wave function $|\Psi(j)|^2$ (with j labeling the distance from an edge) mirrors the hybrid decay of correlations discussed above [Fig. 4(a)]. A numerical fit to the algebraic tail of $|\Psi(j)|^2$ yields $|\Psi(j)|^2 \sim j^{-2\alpha}$ for $\alpha \gtrsim 1$, implying that $|\Psi(j)|^2$ is normalizable, as required for an edge mode [49]. We also note that this algebraic decay of $|\Psi(j)|^2$ is in qualitative agreement with recent calculations for helical Shiba chains [6]. The mass $M(L)$ of the edge modes for $\alpha \gtrsim 1$ exhibits similar hybrid exponential-algebraic behavior (see the Supplemental Material [35]). On the other hand, for $\alpha \lesssim 1$, $M(L)$ becomes nonzero in the limit $L \to \infty$ [Fig. 4(b)].

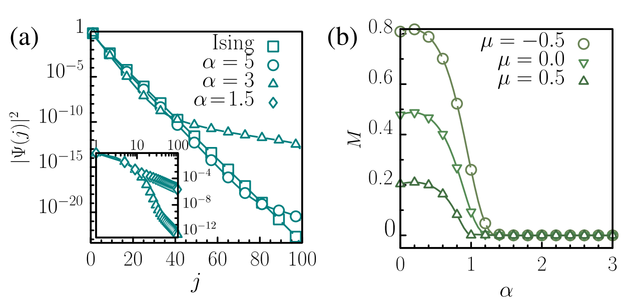

FIG. 4 (color online). Open chain. (a) Wave function $|\Psi(j)|^2$ for $\mu = 0.5$. (b) Mass gap $M(L \to \infty)$ vs α.

Conclusions and outlook.—In this work, we have analyzed a fermionic integrable model with LR pairing, finding gapped phases where correlation functions exhibit purely algebraic or hybrid exponential-algebraic decay, a breaking of the conformal symmetry along gapless lines (directly detected in the dynamics of the von Neumann entropy following a quench, as recently demonstrated numerically in [13]) accompanied by a violation of the area law in gapped phases. It is an exciting prospect to investigate whether some of these results are common to other models with LR interactions, such as Ising-type models, as is currently realized in several labs [8,9].

We thank A. Turner for suggesting the idea of studying the LR Kitaev wire. We thank M. Baranov, P. Calabrese, L. Fidkowski, M. Foss-Feig, Z.-X. Gong, F. Mezzacapo, S. Michalakis, J. Preskill, E. Rico, T. Roscilde, G. Sierra, and L. Taddia for useful discussions. We acknowledge support by the ERC-St Grant ColdSIM (Grant No. 307688), EOARD, and UdS via Labex NIE and IdEX, NSF PFC at JQI, NSF PIF, ARO, Initial Training Network COHERENCE, and computing time at the HPC-UdS.

[1] A. Y. Kitaev, Phys. Usp. **44**, 131 (2001).
[2] E. Majorana, Nuovo Cimento **14**, 171 (1937).
[3] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Rev. Mod. Phys. **80**, 1083 (2008); A. Stern, Nature (London) **464**, 187 (2010).
[4] M. Franz, Nat. Nanotechnol. **8**, 149 (2013).
[5] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Science **336**, 1003 (2012); M. T. Deng, C. L. Yu, G. Y. Huang, M. Larsson, P. Caroff, and H. Q. Xu, Nano Lett. **12**, 6414 (2012); A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum, and H. Shtrikman, Nat. Phys. **8**, 887 (2012); L. P. Rokhinson, X. Liu, and J. Furdyna, Nat. Phys. **8**, 795 (2012); A. D. K. Finck, D. J. Van Harlingen, P. K. Mohseni, K. Jung, and X. Li, Phys. Rev. Lett. **110**, 126406 (2013).
[6] F. Pientka, L. I. Glazman, and F. von Oppen, Phys. Rev. B **88**, 155420 (2013).
[7] J. W. Britton, B. C. Sawyer, A. C. Keith, C.-C. J. Wang, J. K. Freericks, H. Uys, M. J. Biercuk, and J. J. Bollinger, Nature (London) **484**, 489 (2012).
[8] P. Richerme, Z.-X. Gong, A. Lee, C. Senko, J. Smith, M. Foss-Feig, S. Michalakis, A. V. Gorshkov, and C. Monroe, Nature (London) **511**, 198 (2014).

[9] P. Jurcevic, B. P. Lanyon, P. Hauke, C. Hempel, P. Zoller, R. Blatt, and C. F. Roos, Nature (London) **511**, 202 (2014).
[10] C. Schneider, D. Porras, and T. Schaetz, Rep. Prog. Phys. **75**, 024401 (2012).
[11] A. Bermudez, T. Schaetz, and M. B. Plenio, Phys. Rev. Lett. **110**, 110502 (2013); S. Gopalakrishnan, B. L. Lev, and P. M. Goldbart, Phys. Rev. Lett. **107**, 277201 (2011); S. John and J. Wang, Phys. Rev. Lett. **64**, 2418 (1990); E. Shahmoon and G. Kurizki, Phys. Rev. A **87**, 033831 (2013); J. S. Douglas, H. Habibian, A. V. Gorshkov, H. J. Kimble, and D. E. Chang, arXiv:1312.2435.
[12] P. Hauke and L. Tagliacozzo, Phys. Rev. Lett. **111**, 207202 (2013).
[13] J. Schachenmayer, B. P. Lanyon, C. F. Roos, and A. J. Daley, Phys. Rev. X **3**, 031015 (2013).
[14] J. Eisert, M. van den Worm, S. R. Manmana, and M. Kastner, Phys. Rev. Lett. **111**, 260401 (2013).
[15] T. Koffel, M. Lewenstein, and L. Tagliacozzo, Phys. Rev. Lett. **109**, 267203 (2012).
[16] Z.-X. Gong, M. Foss-Feig, S. Michalakis, and A. V. Gorshkov, Phys. Rev. Lett. **113**, 030602 (2014).
[17] Antiperiodic boundary conditions ($a_{j+L} = -a_j$) avoid cancellations between terms like $a_i a_j$ and $a_j a_{i+L}$ and preserve translational invariance.
[18] Different values of Δ/t just rescale the Fermi velocity.
[19] E. Barouch and B. McCoy, Phys. Rev. A **3**, 786 (1971).
[20] P. di Francesco, P. Mathieu, and D. Senechal, *Conformal Field Theory* (Springer, New York, 1997); M. Henkel, *Conformal Invariance and Critical Phenomena* (Springer, New York, 1999).
[21] G. Mussardo, *Statistical Field Theory, An Introduction to Exactly Solved Models in Statistical Physics* (Oxford University, New York, 2010).
[22] X.-L. Deng, D. Porras, and J. I. Cirac, Phys. Rev. A **72**, 063407 (2005).
[23] P. Hauke, F. M. Cucchietti, M. Müller-Hermes, A. Bañuls, J. I. Cirac, and M. Lewenstein, New J. Phys. **12**, 113037 (2010).
[24] D. Peter, S. Müller, S. Wessel, and H. P. Büchler, Phys. Rev. Lett. **109**, 025303 (2012).
[25] V. Nebendahl and W. Dür, Phys. Rev. B **87**, 075413 (2013).
[26] M. L. Wall and L. D. Carr, New J. Phys. **14**, 125015 (2012).
[27] If $L \to \infty$, $f_{k,\alpha} = (1/i)[\mathrm{Li}_\alpha(e^{ik}) - \mathrm{Li}_\alpha(e^{-ik})]$, with $\mathrm{Li}_\alpha(z)$ being a polylogarithm [42,50].
[28] M. B. Hastings, J. Stat. Mech. (2008) L01001.
[29] I. Peschel and T. T. Truong, Z. Phys. B **69**, 385 (1987); T. T. Truong and I. I. Peschel, Z. Phys. B **75**, 119 (1989); I. Peschel, M. Kaulke, and O. Legeza, Ann. Phys. (Berlin) **8**, 153 (1999); I. Peschel, J. Phys. A **36**, L205 (2003); Braz. J. Phys. **42**, 267 (2012).
[30] J. Eisert, M. Cramer, and M. Plenio, Rev. Mod. Phys. **82**, 277 (2010).
[31] L. Amico, R. Fazio, A. Osterloh, and V. Vedral, Rev. Mod. Phys. **80**, 517 (2008).
[32] M. Hastings and T. Koma, Commun. Math. Phys. **265**, 781 (2006).
[33] C. Holzhey, F. Larsen, and F. Wilczek, Nucl. Phys. **B424**, 443 (1994).
[34] P. Calabrese and J. Cardy, J. Stat. Mech. (2004) P06002.
[35] See Supplemental Material athttp://link.aps.org/supplemental/10.1103/PhysRevLett.113.156402, which includes [35–38], for details on analytical calculations of one-body correlation functions, ground-state energy density, and scalings of the entanglement entropy and of the mass gap for the edge modes in an open chain.
[36] M. J. Ablowitz and A. T. S. Fokas, *Complex Variables: Introduction and Applications*, 2nd ed., Cambridge Texts in Applied Mathematics Vol. 35 (Cambridge University Press, Cambridge, England, 2003).
[37] T. Giamarchi, *Quantum Physics in One Dimension*, International Series of Monographs on Physics Vol. 121 (Clarendon, Oxford, 2004).
[38] E. Lieb, T. Schultz, and D. Mattis, Ann. Phys. (N.Y.) **16**, 407 (1961).
[39] J. P. Blaizot and G. Ripka, *Quantum Theory of Finite Systems* (MIT, Cambridge, MA, 1986).
[40] J. Eisert and T. J. Osborne, Phys. Rev. Lett. **97**, 150404 (2006).
[41] The low-lying critical finite-size spectrum that we computed has the same degeneracy pattern as the one in the Ising model, for all α.
[42] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products* (Academic, New York, 2007).
[43] A. A. Belavin, A. M. Polyakov, and A. B. Zamolodchikov, Nucl. Phys. **B241**, 333 (1984).
[44] P. Calabrese and J. Cardy, J. Stat. Mech. (2005) P04010.
[45] For the SR Ising model, contributions at $k = 0$ in Eq. (4) vanish.
[46] L. Lepori *et al.* (unpublished).
[47] This is, for example, at the origin of the behavior of c_{eff}, which is different from zero close to the critical line $\mu = -1$ when $1 < \alpha < 2$.
[48] E. Lieb and D. Robinson, Commun. Math. Phys. **28**, 251 (1972).
[49] E. Lieb, T. Schultz, and D. Mattis, Ann. Phys. (N.Y.) **16**, 407 (1961).
[50] F. W. J. Olver, D. W. Lozier, R. F. Boisvert, and C. W. Clark, *NIST Handbook of Mathematical Functions* (Cambridge University Press, Cambridge, England, 2010); M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1964).