

Topological growing of Laughlin states in synthetic gauge fields

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We suggest a scheme for the preparation of highly correlated Laughlin (LN) states in the presence of synthetic gauge fields, realizing an analogue of the fractional quantum Hall effect in photonic or atomic systems of interacting bosons. It is based on the idea of growing such states by adding weakly interacting composite fermions (CF) along with magnetic flux quanta one-by-one. The topologically protected Thouless pump ("Laughlin's argument") is used to create two localized flux quanta and the resulting hole excitation is subsequently filled by a single boson, which, together with one of the flux quanta forms a CF. Using our protocol, filling $1/2$ LN states can be grown with particle number N increasing linearly in time and strongly suppressed number fluctuations. To demonstrate the feasibility of our scheme, we consider two-dimensional (2D) lattices subject to effective magnetic fields and strong on-site interactions. We present numerical simulations of small lattice systems and discuss also the influence of losses.

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Introduction In recent years topological states of matter [1–8] have attracted a great deal of interest, partly due to their astonishing physical properties (like fractional charge and statistics) but also because of their potential practical relevance for quantum computation [9, 10]. While these exotic phases of matter were first explored in the context of the quantum Hall effect of electrons subject to strong magnetic fields [11, 12], there has been considerable progress recently towards their realization in cold-atom [13–16] as well as photonic [17–23] systems. A particularly attractive feature of such quantum Hall simulators are the comparatively large intrinsic length scales which allow coherent preparation, manipulation and spatially resolved detection of exotic many-body phases and their excitations.

In electronic systems the preparation of topological states of matter relies on quick thermalization and cooling below the many-body gap. While this is already hard to achieve in cold-atom systems (partly due to the small required temperatures), cooling is even less of an option in photonic systems due to the absence of effective thermalization mechanisms. On the other hand, lasers with extremely narrow linewidths allow for a completely different avenue towards preparation of extremely pure quantum states. For instance, it was suggested to use the good coherence properties of lasers to directly excite two (and more) photon LN states in non-linear cavity arrays [24], where the laser plays the role of a coherent pump. However, this approach has the inherent problem of an extremely small multi-photon transition amplitude. While this might be acceptable for small systems of $N = 2, 3$ photons, it makes the preparation of true many-body states with $N \gg 2$ practically impossible. Moreover, the prepared states in this case contain superpositions of different photon-numbers rather than being Fock states.

In this letter we suggest an alternative scheme for the

preparation of topologically ordered states of strongly interacting bosons, and we discuss systems allowing for an implementation of our scheme with state-of-the-art technology. It consists of adiabatically growing such states and makes direct use of the Thouless pump [25] connected to the many-body topological invariant. In the case of quantum Hall physics the latter is realized by local flux insertion in the spirit of Laughlin's argument for the quantization of the Hall conductivity σ_H [26]: Introducing magnetic flux $\phi = 2 \times 2\pi$ in the center of the system produces a quantized outwards Hall current $\sim \sigma_H \partial_t \phi$, leaving behind a hole along with 2 flux quanta, see FIG. 1 (a).

In the next step, the so-created hole can be replenished by a single boson. In view of the composite fermion (CF) picture [27, 28] of the fractional quantum Hall effect, this

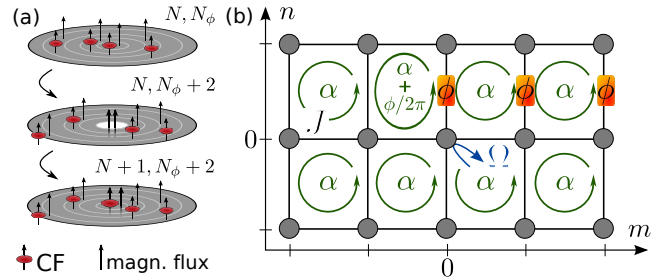


FIG. 1. (Color online) (a) The key idea of our scheme is to grow LN states by introducing weakly interacting CFs into the system. This is achieved by adding magnetic flux (arrows) in the center and replenishing the arising hole by a new boson (red bullet). (b) We consider the Hofstadter-Hubbard model (flux α per plaquette). Additional flux ϕ can be introduced in the center by adiabatically changing the complex phase of the hoppings marked with a box. Furthermore, the central site is assumed to be externally accessible for a coherent drive (Rabi frequency Ω).

refilling step can be interpreted as the addition of a single CF (composed of a bare boson and one flux quantum) into a free orbital of the CF Landau level (LL), using up the remaining flux quantum. To refill such a hole deterministically by a single boson, we consider a coherent pump in the center of the system. Excitations by more than one particle are prohibited by the many-body gap, and the coherent coupling can not decrease the total particle number because the central cavity is empty initially. Thus, our final state has sub-poissonian boson number statistics. A complementary scheme, where holes resulting from boson losses are dynamically refilled in the entire system using single photon pumps, has recently been suggested for photonic systems [29]. Our protocol, in contrast, does not rely on an explicit single photon source to achieve strongly suppressed photon number fluctuations.

A key advantage of our scheme, compared to adiabatic ones, is the ability to grow LN states with a size increasing linearly in time. To reach N particles with given fidelity $1 - \varepsilon$, the protocol has to be carried out sufficiently slow to avoid errors in the repumping protocol. For $\varepsilon \ll 1$ the total required time scales like

$$T \sim \frac{N^{3/2}}{\Delta_{\text{LN}} \varepsilon^{1/2}}, \quad (1)$$

where Δ_{LN} is the bulk many-body gap. In contrast to previously proposed schemes [24, 30, 31], T only grows algebraically with N .

Model We consider a 2D lattice with complex hopping elements (amplitude J) realizing an effective magnetic field, supplemented by Hubbard-type on-site interactions (strength U). This model is illustrated in FIG.1 and can be described by the following Hamiltonian,

$$\begin{aligned} \hat{\mathcal{H}}_{\text{int}} + \hat{\mathcal{H}}_0 &= \frac{U}{2} \sum_{m,n} \hat{a}_{m,n}^\dagger \hat{a}_{m,n} (\hat{a}_{m,n}^\dagger \hat{a}_{m,n} - 1) \\ &- J \sum_{m,n} \left[e^{-i2\pi\alpha n} \hat{a}_{m+1,n}^\dagger \hat{a}_{m,n} + \hat{a}_{m,n+1}^\dagger \hat{a}_{m,n} + \text{h.c.} \right], \end{aligned}$$

where we used Landau gauge and set $\hbar = 1$. There have been numerous suggestions how this Hamiltonian can be implemented in photonic [18, 23, 24, 32, 33], circuit-QED [34–36] or atomic [31, 37] systems, and in the last case this goal has already been achieved [15, 16].

Local flux insertion can most easily be realized by changing the hopping elements from site $(m \geq 0, n = 0)$ to $(m, 1)$ by a factor $e^{i\phi}$, see FIG.1 (b); These links are thus described by

$$\hat{\mathcal{H}}_\phi = -J \sum_{m \geq 0} \left[e^{-i\phi} \hat{a}_{m,1}^\dagger \hat{a}_{m,0} + \text{h.c.} \right], \quad (2)$$

modifying the total magnetic flux through the central plaquette to $\alpha - \phi/2\pi$ (in units of the flux quantum). $\hat{\mathcal{H}}_\phi$ is motivated by recent experiments with photons [18, 23], where the hopping-phases can locally and temporally be

manipulated [38]. Finally to replenish the system with bosons, we place a weak coherent pump ($\Omega \ll 4\pi\alpha J$) in the center,

$$\hat{\mathcal{H}}_\Omega = \Omega e^{-i\omega t} \hat{a}_{0,0}^\dagger + \text{h.c.} \quad (3)$$

In the following we present the details of our scheme, neglecting local boson losses (rate γ) for the moment. We include losses again afterwards in the discussion of the performance of our scheme.

Protocol We begin by discussing the continuum case when the magnetic flux per plaquette $\alpha \ll 1$ is small, allowing us to make use of angular momentum L_z as a conserved quantum number. This case is characterized by a perfectly flat quasihole dispersion, such that holes created in the center remain localized and can reliably be refilled. We will show in a subsequent paragraph how lattice effects modify this simplified picture.

The continuum can be described by LLs, which are eigenstates of $\hat{\mathcal{H}}_0$ in the limit $\alpha \rightarrow 0$ with energies $E_n = (n + 1/2)\omega_c$ ($n = 0, 1, 2, \dots$) and $\omega_c = 4\pi\alpha J$ denoting the cyclotron frequency, see e.g.[28]. The magnetic length is defined as $\ell_B = a/\sqrt{2\pi\alpha}$, where a denotes the lattice constant. In symmetric gauge the single particle states of the LLL are labeled by their angular momentum quantum number $l = 0, 1, 2, \dots$ [28] and we define boson creation operators of these orbitals as \hat{b}_l^\dagger .

For concreteness we here discuss the preparation of LN states at filling $\nu = N/N_\phi = 1/2$, but the generalization to other fillings is straightforward. To create the first excitation, we coherently drive the central cavity with frequency $\omega = \omega_c/2$. When the coherent pump is sufficiently weak, $\Omega \ll \omega_c$, we can neglect excitations of higher LLs and (3) can be projected into the LLL, leading to $\hat{\mathcal{H}}_\Omega \approx [\hat{b}_0^\dagger e^{-i\omega t} \Omega_{\text{eff}}^{(1)} + \text{h.c.}]$ with $\Omega_{\text{eff}}^{(1)} = \Omega\sqrt{\alpha}$. Moreover, having two bosons in the central orbital would cost the energy $\Delta_{\text{LN}} \approx \min(V_0, \omega_c)$, where $V_0 = U\alpha/2$ is given by Haldane's zeroth-order pseudo-potential [39]. Thus, assuming $\Omega_{\text{eff}}^{(1)} \ll \min(V_0, \omega_c)$, no more than one particle can enter the system due to blockade, and we end up with an effective two-level system consisting of the zero and one boson states, $|0\rangle$ and $\hat{b}_0^\dagger|0\rangle$. Then to introduce a single boson, the coherent pump Ω can be switched on for a time $T_\pi = \pi/2\Omega_{\text{eff}}^{(1)}$ corresponding to a π -pulse. For the latter to work, we further require negligible losses $\gamma \ll \Omega_{\text{eff}}^{(1)}$ [40].

Next, we adiabatically introduce two units of magnetic flux into the center of the system. One of them is attached to the boson to form a CF, the second one is needed to keep the filling fraction of CFs constant in the growing scheme. When, for simplicity, the Hamiltonian (2) for flux insertion is replaced by one with a symmetric gauge choice preserving rotational symmetry [41], the initial state $|\Psi_1\rangle = \hat{b}_0^\dagger|0\rangle$ attains two units of angular momentum. Thus we end up in $|\Psi_2\rangle = \hat{b}_2^\dagger|0\rangle$, which has a ring-structure with a hole in its center. Repeating the first step, we can again make use of the blockade

to replenish the hole by exactly one particle. Because the coherent pump couples to the center of the hole, it can not lead to a reduction of the total boson number. Crucially, in contrast to the first step, the new state is not the simple product state $\hat{b}_0^\dagger|\Psi_2\rangle$, which has non-zero interaction energy and is thus blocked. However, there is precisely one zero-energy state with the correct total angular momentum $L_z = 2$, namely the $N = 2$ LN state $|\text{LN}, 2\rangle$, which is coupled to $|\Psi_2\rangle$ by a Rabi frequency reduced by a Franck-Condon factor (FCF), $\Omega_{\text{eff}}^{(2)}/\Omega = \langle \text{LN}, 2 | \hat{b}_0^\dagger | \Psi_2 \rangle \sqrt{\alpha}$.

Having established our protocol for two bosons, the extension to N -particle LN states $|\text{LN}, N\rangle$ is straightforward. In this case, local flux insertion is used to create two quasiholes in the center $|2\text{qh}, N-1\rangle$, which are refilled by the coherent pump to prepare $|\text{LN}, N\rangle$. The corresponding transition amplitude $\Omega_{\text{eff}}^{(N)}$ is reduced by a many-body FCF,

$$\Omega_{\text{eff}}^{(N)}/\Omega = \sqrt{\alpha} \langle \text{LN}, N | \hat{b}_0^\dagger | 2\text{qh}, N-1 \rangle, \quad (4)$$

which even for large N takes a non-zero value. Using exact diagonalization (ED) of small systems ($N = 1, \dots, 9$) we find that $\Omega_{\text{eff}}^{(N)}$ is nearly constant as a function of N and we extrapolate $\Omega_{\text{eff}}^{(\infty)} \approx 0.70 \Omega \sqrt{\alpha}$. Thus our pump protocol works equally efficient for large and small boson numbers.

A natural explanation why highly correlated many-body states can be grown in the relatively simple fashion described above is provided by the composite fermion picture: LN states are separable (Slater determinant) states of non-interacting CFs filling the CF-LLL [27]. Thus, introducing CFs one-by-one into the orbitals of this LLL, LN states can easily be grown.

To ensure a sizable cyclotron gap ω_c , a not too small flux per plaquette α is desirable, where lattice effects become important. We will now study this regime, which is also of great experimental relevance [15, 16, 23]. The spectrum of the Hamiltonian $\hat{\mathcal{H}}_0(\alpha)$ is the famous Hofstadter butterfly [42], consisting of a self-similar structure of magnetic sub-bands. When interactions are taken into account, LN-type states can still be identified at filling $\nu = 1/2$ [31, 37].

The basic ideas of our protocol directly carry over to the lattice case. Because the many-body Chern number is strictly quantized, Laughlin's argument shows that a hole excitation can still be created by local flux insertion. However, due to the formation of magnetic sub-bands, such a quasihole will propagate away from the center. This leaves us only a restricted time to refill the defect, and, more dramatically, provides a decoherence mechanism leading to a reduced efficiency of repumping. To circumvent this problem, we introduce a trap for quasiholes. A static, repulsive potential of the form

$$\hat{\mathcal{H}}_{\text{pot}} = \sum_{m,n} \frac{g}{\sqrt{2\pi\ell_B/a}} e^{-(m^2+n^2)a^2/2\ell_B^2} \hat{a}_{m,n}^\dagger \hat{a}_{m,n} \quad (5)$$

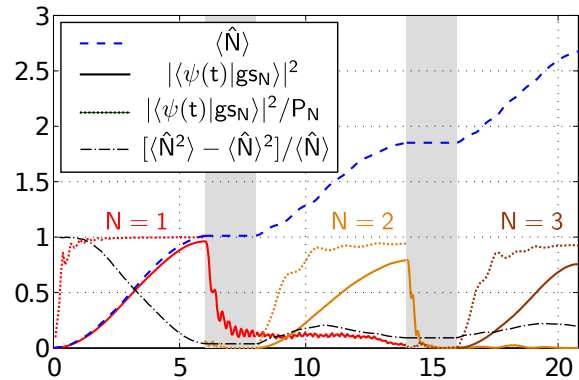


FIG. 2. (Color online) Simulation of the full protocol on a C_{60} buckyball for $U = 10J$ and including the static potential (5) with $g = J$. We start from vacuum and $N_\phi = 0$ flux quanta. Then the coherent pump Eq.(3) is switched on for a time $T_\Omega = 6\pi/\Omega$ (with $\Omega = 0.05J$) and one boson is inserted with an overlap (solid red) [conditioned on $N = 1$ (dotted red)] close to one to the target $N = 1$ ground state. The driving frequency ω is chosen to be resonant on the transition from $N = 0$ to $N = 1$ ground state. After introducing two more flux quanta in a time $2 \times 20\pi/J$ the whole protocol is repeated and we finally arrive close to a three particle LN-type ground state.

is sufficient for a gapped ground state at any point in the protocol. An alternative would be to include carefully chosen long-range hoppings leading to a completely flat band [43].

In the following we use ED to simulate our protocol for small systems. To get rid of boundary effects, which can be pronounced in small systems, we consider a spherical geometry [44] and take into account lattice effects by using a buckyball-type lattice. The hopping elements on all links have amplitude J and their phases were chosen such that the flux per plaquette is α . Because the total flux N_ϕ is integer quantized, it holds $\alpha = N_\phi/N_p$ with $N_p = 32$ being the number of plaquettes. We checked numerically (using ED) that for the values of $\alpha \leq 0.2$ used in this paper there are gapped LN-type ground states, provided that the condition $N_\phi = 2(N-1)$ for $\nu = 1/2$ LN states on a sphere is fulfilled. We find gaps of the order $\Delta_{\text{LN}} \approx 0.1J$, as predicted for a square lattice [31, 37].

To describe the effect of local flux insertion $N_\phi \rightarrow N_\phi + \phi/2\pi$ we slightly increase $\alpha \rightarrow \alpha + \phi/(2\pi N_p)$ everywhere, except on the central plaquette where $\alpha \rightarrow \alpha - (1 - 1/N_p)\phi/2\pi$ changes by $-\phi/2\pi$ in thermodynamic limit (i.e. for $N_p \rightarrow \infty$). Starting from an incompressible LN-type ground state, we checked numerically that the correct number of low-lying quasihole states is obtained, and that they can be gapped out by the potential Eq.(5) (see Supplementary Material for more details).

In FIG.2 we present a numerical simulation of our full protocol on the C_{60} buckyball lattice. We find that the overlaps of the prepared states to the targeted N par-

ticle ground states $|\text{gs}_N\rangle$ is close to one after all steps, and the overlaps conditioned on having the correct particle number N (occurring with probability P_N) are even larger. The time required for each cycle is of the order of $2\pi/\Delta_{\text{LN}} \approx 60/J$. At the end of the protocol, the $N = 3$ boson ground state at $N_\phi = 4$ is prepared with high fidelity, which carries the signatures of a LN-type state. Importantly the particle number fluctuations after a completed cycle are strongly suppressed $[\langle \hat{N}^2 \rangle - \langle \hat{N} \rangle^2] / \langle \hat{N} \rangle \ll 1$.

In our simulations we neglected edge effects and bulk losses. The latter result in a finite boson life-time, such that in the growing scheme the mean density $\rho(r)$ decays with the distance r from the center. In continuum we find $\rho(r) \approx \frac{1}{4\pi\ell_B^2} \exp(-\gamma T_0 \frac{r^2}{4\ell_B^2})$, with T_0 being the duration of a single step of the protocol. In a forthcoming publication [45] we study larger systems using a simplified model of non-interacting CFs on a lattice and show that our protocol still works when edge-effects are taken into account.

In FIG.2 we observe that the fidelity $\mathcal{F}_N = |\langle \psi(t) | \text{gs}_N \rangle|$ for preparation of the N -particle LN-type ground state is limited, mostly by the inefficiency of the pump. High fidelity, however, is an indispensable requisite for measuring e.g. braiding phases of elementary excitations, which play a central role for topological quantum computation [10]. Taking into account couplings between low-energy states of the N and $N + 1$ boson sectors, induced by the coherent pump (3), we find the following expression for the fidelity,

$$\mathcal{F}_N \sim \exp\left(-\left(\frac{\Lambda^2}{\Delta_{\text{LN}}^2 T_0^2} + \gamma T_0 \frac{N}{2}\right) \frac{N}{2}\right). \quad (6)$$

Here Λ is a numerical parameter determined by non-universal FCFs, and from FIG.2 we estimate $\Lambda \approx 10$ [46]. The second term in Eq.(6) describes boson loss, whereas the first term takes into account imperfections of the blockade in the repumping process with rates scaling like $\sim (\Omega_{\text{eff}}/\Delta_{\text{LN}})^2$. In Eq.(6) we neglected fidelity losses from flux insertion, which according to the Landau-Zener formula [47] yield only double exponential corrections, however, and within this approximation $T_0 \approx T_\pi = \pi/2\Omega_{\text{eff}}$.

In Eq.(6) we observe a competition between losses $\sim T_0$ and errors of the pump $\sim 1/T_0^2$. Thus, for a target fidelity $\mathcal{F}_N = 1 - \varepsilon$, only LN states of a restricted number of bosons $N \leq N_{\text{max}}$ can be grown,

$$N_{\text{max}} = 1.365 \varepsilon^{3/5} \left(\frac{\Delta_{\text{LN}}}{\Lambda\gamma}\right)^{2/5}. \quad (7)$$

To do so, a time $T = N_{\text{max}} T_0 = 1.22 N_{\text{max}}^{3/2} \varepsilon^{-1/2} \Lambda / \Delta_{\text{LN}}$ is required, which yields Eq.(1).

Experimental realization In photonic cavity arrays [18, 23, 24, 32, 33], the main experimental challenges for realizing our protocol are the required large interactions $U \gtrsim J$ and small losses $\gamma \ll \Delta_{\text{LN}}/N^{5/2}$. Strong

non-linearities can be realized e.g. by placing single atoms into the cavities [32] or coupling them to quantum dots [23] or Rydberg gases [23, 48, 49]. Most promising, we believe, are circuit-QED systems, where loss-rates $\gamma = 10\text{kHz}$ have already been achieved [50] and in which the strong coupling regime can be reached with $U = 100\text{MHz} = 10J$ [36]. The latter gives $\Delta_{\text{LN}} \approx 0.1J \approx 1\text{MHz}$ which corresponds to $\Delta_{\text{LN}}/2\pi \approx 17\gamma$. To overcome losses, an array of multiple flux and photon pumps could be envisioned.

In ultra cold atom systems [15, 16] on the other hand, large interactions U and negligible decay γ are readily available [51]. Local detection and addressing of single atoms has also been demonstrated [52–55]. To realize the coherent pump Eq.(3) one could e.g. use an atomic BEC in an internal state $|\downarrow\rangle$ as a reservoir, and couple it to the system in the internal state $|\uparrow\rangle$ by microwave pulses. Supplementing this setup by a focused laser beam, it has been shown that local addressing is possible [55]. We believe that currently the biggest issue would be the realization of local flux insertion. However, an alternative way of creating quasiholes would be to introduce a focused laser-beam close to the edge of the system, the intensity of which is adiabatically increased [56]. Also in this configuration atoms are only added along with flux quanta, which is the essence of our scheme.

Summary & Outlook We proposed a scheme for the preparation of highly correlated LN states of bosons in artificial gauge fields. LN states can be understood in terms of weakly interacting CFs, and our protocol is based on the idea of adiabatically growing non-correlated states of the latter. We demonstrated that this can be achieved by first creating LN quasihole excitations which are subsequently refilled with bosons. Importantly, our protocol only requires a preparation time scaling slightly faster than linear with system-size.

Our scheme is not restricted to the preparation of LN states of bosons. For example, we expect that the $\nu = 1$ bosonic Moore-Read Pfaffian [5, 57, 58] supporting non-Abelian topological order, can also be grown using our technique. Moreover, preparing bosons in higher LLs opens the possibility to simulate exotic Haldane pseudopotentials, mimicking the effect of long-range interactions without the need to implement these in first place. We also expect that our scheme can be adapted for the preparation of fractional quantum Hall states of fermions.

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SUPPLEMENTARY: LAUGHLIN STATES ON THE BUCKYBALL LATTICE

To simulate fractional Chern insulators – i.e. the lattice analogues of Laughlin states – on a finite lattice system without edges, we consider bosons hopping on the bonds between the 60 sites of a buckyball. All hopping elements are assumed to be of magnitude J and their phases are chosen in such a way that in total an integer amount N_ϕ of flux quanta pierce the surface, with homogeneous flux per plaquette α (in units of the flux quantum). This is a simple generalization of the sphere surrounding a magnetic monopole which was introduced by Haldane [44].

Because in the spherical geometry – unlike in the case of a torus – Chern numbers can not readily be calculated from geometric Berry phases, we need to chose an alternative approach to identify Laughlin (LN) type ground states. To this end we adiabatically introduce magnetic flux through a single plaquette (say at the north pole), thereby increasing the charge of the fictitious magnetic monopole in the center of the sphere. This corresponds to the flux insertion described in the main text. As a consequence, an outwards Hall current pointing from north to south pole is generated, which is proportional to the Chern number of the many body state.

In FIG.3 we show the flux-insertion spectra (i.e. the eigenenergies as a function of ϕ) for $N = 3$ bosons on the buckyball lattice. In (a) we did not include the trapping potential Eq.(5) from the main text, and thus for $N_\phi = 4$ we expect an incompressible LN-type ground state from the condition $N_\phi = 2(N - 1)$ for $\nu = 1/2$ LN states on a sphere. Indeed, we observe a ground state gap of the order $\Delta_{\text{LN}} \approx 0.1J$ in (a) as predicted for a square lattice [31, 37]. Moreover, for $\phi = 2\pi$ and 4π the correct counting of (nearly degenerate) quasihole states is obtained, supporting our assumption that the ground state is in the LN universality class.

In FIG. 3 (b) the trapping potential Eq.(5) from the main text is included and the quasihole degeneracy is split. For all values of the additional magnetic flux ϕ a gapped ground state is observed, and by calculating the corresponding density profiles we checked that in the flux insertion the expected Hall-current corresponding to a Chern number $C = 1/2$ is generated. This moreover shows that our intuitive picture of the ground state – consisting of a quasihole trapped by the potential – applies. Finally, by adiabatically increasing g from $g = 0$ in (a) to $g = J$ in (b) we checked that the ground state gap does not close at $\phi = 0$ and its topological properties are thus unchanged by the potential Eq.(5) from the main text.

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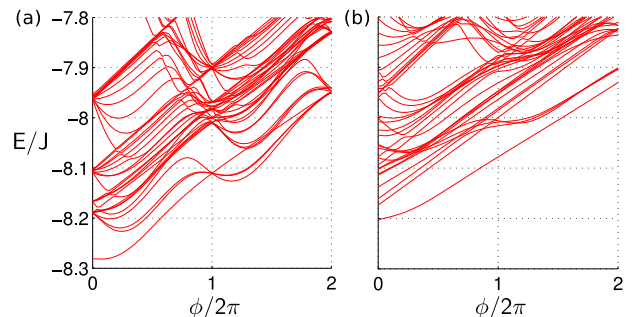


FIG. 3. (Color online) Many-body spectra for $N = 3$ bosons on a C_{60} buckyball threaded by $N_\phi = 4 + \phi/2\pi$ flux quanta. (a) Without trapping potential $g = 0$, a gapped LN-type state is observed for $\phi = 0$, which turns into a manifold of degenerate quasihole states for $\phi \geq 2\pi$. (b) When a weak trapping potential Eq.(5) is switched on (here $g = J$), the ground state is gapped for all ϕ . $U = 10J$ was used in both cases and for $\phi = 0$ ($\phi = 4\pi$) we have $\alpha = 0.125$ ($\alpha = 0.1875$).

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