# Thermal radiation as a probe of one-dimensional electron liquids

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Motivated by recent developments in the field of plasmonics, we develop the theory of radiation from one-dimensional electron liquids, showing that the spectrum of thermal radiation emitted from the system exhibits signatures of non-Fermi liquid behavior. We derive a multipole expansion for the radiation based on the Tomonaga-Luttinger liquid model. While the dipole radiation pattern is determined by the conductivity of the system, we demonstrate that the quadrupole radiation can reveal important features of the quantum liquid, such as the Luttinger parameter. Radiation offers a probe of the interactions of the system, including Mott physics as well as nonlinear Luttinger liquid behavior. We show that these effects can be probed in current experiments on effectively one-dimensional electron liquids, such as carbon nanotubes.

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## I. INTRODUCTION

Plasmons are fundamental excitations of electron liquids that emerge from the coupling between the collective motion of charge and the electromagnetic field. In conventional metals in three dimensions, plasmons are gapped to high frequencies, typically above the ultraviolet. However, plasmon excitations at the surface of metals [1] (surface plasmons) or in low-dimensional electron systems such as semiconductor heterostructures, graphene or carbon nanotubes become gapless [2–5]. As a result, such plasmons can be studied in more experimentally accessible frequency ranges—from the microwave to the optical domain [6–14]. The exploitation of these low-dimensional plasmons for technological applications including biosensing, optical communication, and information processing is a burgeoning field of research [15–17].

Plasmons play a central role in the emergent physics of low-dimensional electron systems. In particular, one-dimensional (1D) electron liquids, where electron-electron interactions play a crucial role, represent an important exception to Landau's Fermi liquid theory [18–20]. A conventional formalism to treat such 1D systems is the Tomonaga-Luttinger liquid (TLL) framework in which the excitations are described by free bosons with an acousticlike spectrum. For repulsive interactions, the velocity of these excitations $v$ is renormalized from the Fermi velocity $v_F$, with $v > v_F$. However, this picture is not exact, and short distance behavior and nonlinearities can give rise to physics beyond the TLL paradigm [21–25]. Experimentally, the most common manner in which 1D systems are probed is by electron transport. However, transport measurements are often dominated by the properties of Fermi-liquid leads and DC measurements do not directly reveal interaction effects [26]. More generally, direct signatures of non-Fermi liquid behavior remain challenging to observe experimentally [27].

In this work, we demonstrate that thermal radiation in the optical range can serve as a novel probe of non-Fermi liquid behavior in 1D systems. Within the TTL framework, we develop the theory of radiation from a generic 1D electron liquid, in terms of a multipole expansion in the small parameter $v/c$, valid for long systems (i.e., much longer than the wavelength of typical radiation). We show that the dipole radiation offers an alternative probe of the AC conductivity, $\sigma(\omega)$ [28]. One of the key predictions of the TLL model is the renormalization of $v$ due to interactions. We demonstrate how $v$ can be gleaned from the thermal radiation, by considering both dipole and quadrupole emission. Our work also shows how radiation offers a probe of more subtle effects. We calculate the effect of Mott insulating behavior on the radiative properties of a 1D liquid. Additionally, we show that the quadrupole radiation field reveals subtle signatures of nonlinear TLL effects. Finally, our theory allows us to make detailed predictions of these effects for carbon nanotubes.

Carbon nanotubes are well suited to studies of thermal radiation [29]. Their large ($\sim$eV) bandwidths allow the Luttinger liquid regime to be probed at room temperature. Although the radiation from carbon nanotubes has been studied in the context of single electron physics [30] and quantized plasmons [27,31], the implications of non-Fermi liquid behavior on the radiation has, to our knowledge, not been explored.

## II. LUTTINGER LIQUID

We consider the radiation from a long system [see Fig. 1(a)], such that the length $L$ of the system greatly exceeds the characteristic wavelength $1/q$ of the excitations of the 1D liquid. This condition gives $L \gg \hbar v/k_B T$ and ensures that finite size quantization is not reflected in the emission spectrum [31]. Electromagnetic fields couple to matter via

$$H_{\text{int}} = -e \int d\mathbf{r}\, \mathbf{J}(\mathbf{r},t) \cdot \mathbf{A}_{\text{rad}}(\mathbf{r},t), \tag{1}$$

 

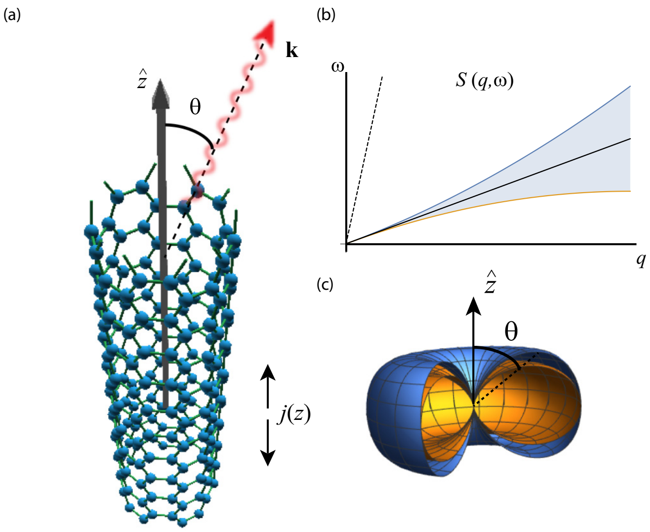


FIG. 1. (a) Thermal radiation from an armchair carbon nanotube. (b) Shaded area indicates regions in the $(q, \omega)$ plane for which the dynamic structure factor $S(q, \omega)$ is nonzero for (i) the Tomonaga-Luttinger model at zero temperature (black line) and (ii) an electron gas at zero temperature (shaded region). The dotted line shows the relation $\omega = cq/\cos\theta$, which is the condition for momentum to be conserved. (c) The angular dependence of the radiated power for a pure dipole (orange inner shell) and dipole with a quadrupole correction (blue outer shell) where the scale of the quadrupole correction has been exaggerated.

where $\mathbf{A}_{\rm rad}$ is the vector potential corresponding to the radiation field and $\mathbf{J}(\mathbf{r}, t)$ is the three-dimensional electron number current. Specializing to the case of a 1D electron liquid oriented along the $z$ axis, we introduce the 1D electron number current operator $j_{1D}(z, t)$ whose Fourier transform is given by

$$j_{1D,\mathbf{k}} = \int dz\, e^{-i|\mathbf{k}|z\cos\theta} j_{1D}(z, t), \tag{2}$$

where $\mathbf{k}$ is the wave number of the emitted photon and $\theta$ is the angle between $\mathbf{k}$ and the $z$ axis, see Fig. 1. The spontaneous emission rate involving a transition between the many-body states $|n\rangle$ and $|m\rangle$ with energies $\hbar\omega_{n,m}$ is given by Fermi's golden rule,

$$\frac{d^2\dot{N}_{n\to m}}{d\omega\, d\Omega} = \frac{\pi e^2}{\hbar\varepsilon_0}\frac{\omega^2}{(2\pi c)^3}\,|\langle m|j_{\mathbf{k}}|n\rangle|^2|\hat{z}\cdot\hat{\varepsilon}^*|^2, \\ \times\delta(\omega_n-\omega_m-\omega), \tag{3}$$

where $\hat{\varepsilon}$ is the photon's polarization. The total rate at which photons are emitted, $\dot{N}$, can be obtained by summing Eq. (3) over all initial and final states. The result can be written in terms of the (current-current) structure factor $S(q, -\omega)$ [32]. We obtain

$$\frac{1}{L}\frac{d^2\dot{N}}{d\omega\, d\Omega} = 2\pi\alpha c\frac{\omega^2}{(2\pi c)^3}\frac{S\left(\frac{\omega}{c}\cos\theta, -\omega\right)}{\omega}\sin^2\theta, \tag{4}$$

where $c$ is the speed of light. This expression contains the following factors: the fine structure constant $\alpha \approx 1/137$, a photon phase space factor $\propto\omega^2$, and $S(q, -\omega)$, which for $\omega > 0$ is related to the loss spectrum of the system [33]. In deriving Eq. (4), we have assumed that the coherence length associated with $S(q, -\omega)$ is much smaller than the length $L$ of the system. For a system in thermal equilibrium,

$$S(q, -\omega) = \frac{2\chi''_{jj}(q, -\omega)}{e^{\beta\hbar\omega}-1}, \tag{5}$$

where $\chi''_{jj}(q, \omega)$ is the imaginary part of the current-current correlator and $\beta = 1/k_BT$. Equation (5) follows from detailed balance, i.e., $S(q, \omega) = e^{\beta\hbar\omega}S(q, -\omega)$.

The spatial dependence of the emitted radiation profile is obtained by expanding $\chi''_{jj}(q_\omega, \omega)$ in the small parameter $v/c$, which effectively corresponds to the small $q$ expansion

$$\chi''_{jj}(q, -\omega) = \chi''_{jj}(0, -\omega) + \frac{1}{2!}\frac{\partial^2\chi''_{jj}(0, -\omega)}{\partial q^2}q^2 + \ldots. \tag{6}$$

Only even powers of $q$ are allowed for a system with inversion and time-reversal symmetries. Due to the factor $|\hat{z}\cdot\varepsilon|^2 = \sin^2\theta$ in Eq. (4), $\chi''_{jj}(0, -\omega)$ controls the dipole radiation. Since $q \propto \cos\theta$ in Eq. (4), the second term in Eq. (6) describes quadrupole radiation, which is also studied in this work.

The TLL model describes the low energy properties of a strongly correlated degenerate electron system. For a noninteracting electron gas, the low-energy excitations are particle-hole excitations which move at the Fermi velocity $v_F$. For short-ranged interactions, the charge excitations of the system can be described by the TLL Hamiltonian

$$H_0 = \frac{\hbar v}{2}\int dx\left[K(\partial_x\theta)^2 + \frac{1}{K}(\partial_x\phi)^2\right], \tag{7}$$

where the charge density is given by $\rho = n + \partial_x\phi/\sqrt{\pi}$ while the current is $j = v\partial_x\theta/\sqrt{\pi}$ [19]. This Hamiltonian can be obtained by bosonizing the electron operators. The quantized form of the charge excitations of the LL are referred to as *plasmons*. The many-body states appearing in Eq. (3) can be characterized by the occupation number of the corresponding modes. Equation (1) describes the coupling of these plasmons to photons.

The propagation speed of the plasmons differs from $v_F$ which can be deduced from the Fermi velocity of graphene (for the case of armchair carbon nanotubes near half filling, $v_F \approx 1.0\times10^6$ m/sec). The quantity $K = v_F/v$ is a measure of the strength of the interactions. The noninteracting limit corresponds to $K = 1$. For a 1D system with transverse size $R$, $K = [1 + (2Ne^2/\pi\hbar v_F)\ln(R_s/R)]^{-1/2}$, where $R_s$ is the screening length of the Coulomb interaction [3,34]. The parameter $N$ is the number of channels, e.g., in carbon nanotubes $N = 4$, arising from two spin and valley degrees of freedom, and typically $K \approx 0.2$–$0.3$.

Given that it is the current that couples to the radiation field, the structure factor corresponding to the retarded current-current correlator $\chi_{jj}$ controls the emission spectrum of the system. The current-current correlator of a Luttinger liquid is given by

$$\chi_{jj}(q, \omega) = \frac{v}{2\pi}\left(\frac{\omega}{\omega - vq + i/\tau} + \frac{\omega}{\omega + vq + i/\tau}\right), \tag{8}$$

where $\tau^{-1}$ is the plasmon damping rate, which will be discussed below. Equation (8) can be obtained from the density-density correlator $\chi_{\rho\rho}$ and the continuity equation in the form

$q^2\chi_{jj} = \omega^2\chi_{\rho\rho}$ [20]. The two terms in Eq. (8) represent right- and left-moving charged excitations moving with a speed $v$.

Taking the imaginary part of Eq. (8) yields $\chi''_{jj}(q,-\omega) \propto \sum_{\pm}\delta(\omega \pm vq)$ (with $\tau \to \infty$). The on-shell condition $\omega = \pm qv$ is incompatible with momentum conservation, $\omega = cq/\cos\theta$ [see Fig. 1(a)]. We are apparently led to the conclusion that the TLL model does not admit radiation. The vanishing of the off-shell spectral weight for the TLL model can ultimately can be traced to the integrability of the model (see, e.g., Ref. [35]). Integrable models have the property that the excitations of the system are infinitely long lived.

In a real quantum liquid, the lifetimes of the plasmonic excitations are rendered finite due to corrections not accounted for in the TLL model as well as coupling to the external environment. The latter may include impurities, phonons, and lossy materials [27]. Here, we account for these effects by considering an open Luttinger liquid which is coupled to a bath of harmonic oscillators [36,37]. It is found that the damping rate $\tau^{-1}(q,\omega)$ depends on the spectral function $J(q,\omega)$ which is a function of the coupling to the external modes and the density of these modes [36]. In particular, we have that $\mathrm{Re}\,\tau^{-1}(q,\omega) \propto J(q,\omega)/\omega$ for $\omega > 0$. The case of $J(q,\omega) \propto |\omega|$ corresponds to an Ohmic bath and is characterized by an $\omega$-independent damping rate. The dipole radiation is controlled by $J(0,\omega)$. We take $J(0,\omega) \propto \omega^{\alpha+1}$ where $\alpha$ is not necessarily an integer. This gives $\tau^{-1}(0,\omega) = (\omega/\omega_*)^{\alpha}/\tau_0$. The case $\alpha = 0$ corresponds to the Ohmic case. The case of a system coupled to three-dimensional acoustic phonons described by $\alpha = 2$ is shown in Fig. 2(b) [37].

## III. THERMAL RADIATION

*Dipole contribution.* The dipole contribution to the radiation is determined by the conductance since, according to the Kubo formula, the first term of the expansion in Eq. (6) can be written $\lim_{q\to 0}\chi''_{jj}(q,-\omega) = \frac{\hbar\omega}{e^2}\mathrm{Re}\,\sigma(\omega)$. The dipole spectrum can thus be written as

$$\frac{1}{L}\frac{d^2\dot{N}_{\mathrm{dip}}}{d\omega\,d\Omega} = \frac{1}{8\pi^3\varepsilon_0 c^3}\frac{\omega^2\,\mathrm{Re}\,\sigma(\omega)}{e^{\beta\hbar\omega}-1}\sin^2\theta, \tag{9}$$

where $\dot{N}_{\mathrm{dip}}$ is the rate at which photons are emitted in the dipole channel. This result demonstrates that the spectrum of emitted radiation offers a powerful probe of $\sigma(\omega)$. For the Ohmic case with $\alpha = 0$, Eq. (8) corresponds to a Drude conductivity for $\sigma(\omega)$. The emission spectrum (integrated over all solid angles) is given by

$$\frac{1}{L}\frac{d\dot{N}_{\mathrm{dip}}}{d\omega} = \frac{4\alpha v}{3\pi^2c^2}\left[\frac{\omega^2\tau(0,\omega)}{1+(\omega\tau(0,\omega))^2}\right]\frac{1}{e^{\beta\hbar\omega}-1}. \tag{10}$$

Figure 2 shows the spectrum of the dipole emission for both Ohmic and non-Ohmic baths.

For the remainder of the paper, we focus on the case of an Ohmic bath with $\tau(0,\omega) = \tau_0$. The salient features of the spectrum depend on the ratio $\eta = k_BT\tau_0/\hbar$. In the regime $\eta < 1$, the spectrum mimics that of a classical black body and is characterized by a lack of quantum coherence. For $\eta \ll 1$,

$$\frac{\dot{N}_{\mathrm{dip}}}{L} = \frac{8\zeta(3)\alpha}{3\pi^2}\frac{v\tau_0(k_BT)^3}{c^2\hbar^3} \tag{11}$$

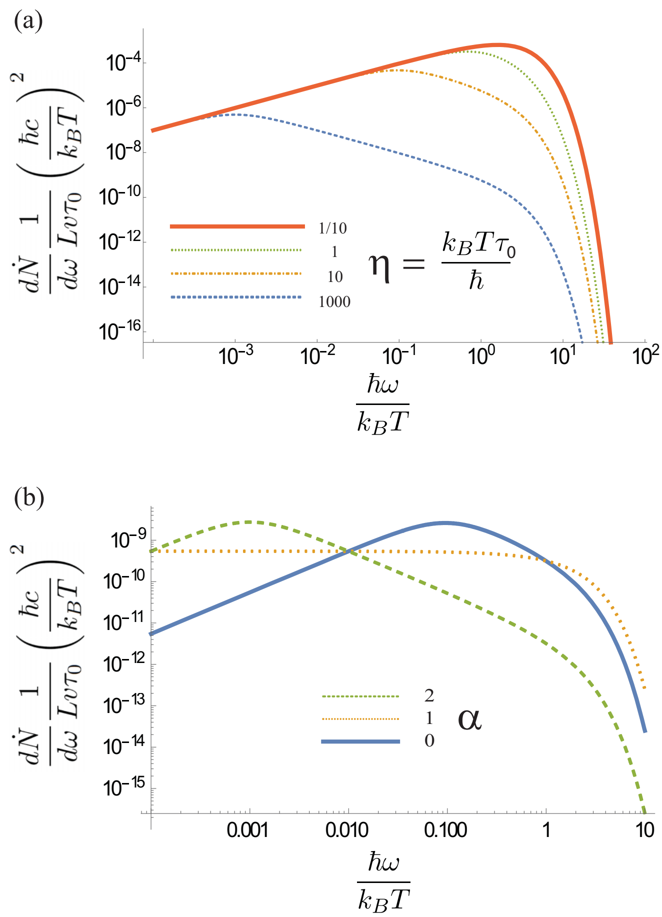


FIG. 2. (a) Log-log plot of the spectrum of photon emission in the dipole channel for a TLL coupled to an Ohmic bath for various values of $\eta = k_BT\tau_0/\hbar$. (b) Same as (a) with Ohmic ($\alpha = 0$) and super-Ohmic baths ($\alpha = 1, 2$) with $1/\tau(0,\omega) = (\omega/\omega_*)^{\alpha}/\tau_0$, $\hbar\omega_* = k_BT/100$, and $\eta = 10$.

and the emission spectrum has the black-body form for a hypothetical cylinder of radius $\alpha v\tau_0$ with an emissivity $\varepsilon = 0.04$. This physics may be relevant to the observation of thermal black-body radiation seen in metallic carbon nanotubes [29].

In the opposite regime $\eta \gg 1$, the TLL bosons are phase coherent and one expects the strongest signatures of non-Fermi liquid behavior in the radiation. In this case, the total rate at which photons are emitted goes as $\dot{N}_{\mathrm{dip}} \propto T/\tau_0$, and the spectrum $d\dot{N}_{\mathrm{dip}}/d\omega$ achieves its maximum near $\omega = \tau_0^{-1}$. The rate at which energy (per unit length) is lost to radiation in this regime is

$$\frac{\dot{P}_{\mathrm{dip}}}{L} = \frac{2\alpha v(k_BT)^2}{9c^2\hbar\tau_0}. \tag{12}$$

This can be compared with the total power dissipated per unit length, which is given by $\pi T^2/6\hbar v\tau_0$ [19]. Thus, the ratio of power emitted as radiation to the total dissipated power is roughly $\alpha(v/c)^2$, which is consistent with our assumption that only a small portion of dissipation is due to radiation.

Drude behavior in $\sigma(\omega)$ itself should not be taken as evidence of TLL behavior. Although Eq. (8) formally depends on $v$, one finds that if $\tau_0$ is independent of energy, then $\sigma(\omega)$ becomes insensitive to interactions and the renormalization effects of $v$. This follows from the $f$-sum rule $\int_0^\infty \mathrm{Re}\,\sigma(\omega)\,d\omega = \pi e^2 n/2m$. Consequently, the Luttinger parameter cannot be extracted from the dipole radiation alone. As we will show below, $v$ can be extracted if the quadrupole emission is also known. The observation of a plasmon velocity $v$ with $v > v_F$ would be a signature of TLL physics.

*Quadrupole contribution.* The quadrupole contribution to the radiation can be gleaned from the $q^2$ term in the expansion of $\chi''_{jj}(q,\omega)$ [see Eq. (6)]. The correction to the distribution of emitted radiation arising from the quadrupole channel is given by

$$\frac{1}{L}\frac{d\dot{N}_{\mathrm{quad}}}{d\omega} = \frac{4\alpha v^3}{15\pi^2 c^4 \tau_0}\left[\frac{3(\omega\tau_0)^6 - (\omega\tau_0)^4}{\left(1+(\omega\tau_0)^2\right)^3}\right]\frac{1}{e^{\beta\hbar\omega}-1}. \quad (13)$$

The correction to the radiated power arising from the quadrupole radiation goes as $\dot{N}_{\mathrm{quad}} \propto T$. Relative to Eq. (9), Eq. (13) is suppressed by a factor $(v/c)^2$. This relation allows for a determination of $v$ and thus gives $K$ provided $v_F$ is known.

*Nonquadratic perturbations to $H_0$.* Perturbations to $H_0$ involving sine-Gordon terms as well as nonlinearities in the Luttinger model give rise to perturbations to $\chi''_{jj}(k,\omega)$. The density dependence of the Luttinger parameters $v$ and $K$ give rise to scattering among the bosonic quasiparticles [21]. The interaction between right- and left-moving bosons is described by

$$H_\gamma = \gamma \int dx\left[(\partial_x\varphi_L)^2\partial_x\varphi_R - (\partial_x\varphi_R)^2\partial_x\varphi_L\right], \quad (14)$$

where the chiral fields $\varphi_{R/L} = \frac{1}{2}(\theta \mp \phi)$. The parameter

$$\gamma \propto \hbar[\partial_n(vK) - \partial_n(v/K)] \quad (15)$$

and is thus controlled by the density dependence of the Luttinger parameters $v$ and $K$; as such, $\gamma$ vanishes for the standard Luttinger liquid case in which $v$ and $K$ are independent of electron density $n$.

The scattering processes arising from the terms in $H_\gamma$ contribute to $\tau(\omega)$ giving rise to a correction to the conductivity $\delta\sigma(\omega) \sim \gamma^4\omega^3 + O(\omega^5)$ for the case of a system with broken Galilean invariance [38]. This term contributes to the dipole radiation and corresponds to an integrated flux $\dot{N}_{\mathrm{dip}}$ which scales as $T^6$. A more significant contribution to the radiation arises as a correction to the quadrupole term (13). Computing the corrections to $\chi''_{jj}(q,\omega)$ which arise from $H_\gamma$ at 1-loop, we find

$$\delta\chi''_{jj}(q,\omega) \sim \frac{\gamma^2 q^2}{\hbar^2 v}, \quad (16)$$

for $\omega \gg vq$. This result may also be obtained from the continuity equation and the analogous result for the density-density correlator [23,24]. Physically, $\delta\chi''_{jj}(q,\omega)$ arises from the relaxation of left- and right-moving bosons such that their momenta nearly cancel. The best chance to observe the effects of Eq. (16) are in systems with very small effective electron mass.

At half filling, carbon nanotubes can exhibit a Mott gap [39]. In the Luttinger liquid framework, a careful microscopic description of the Coulomb interaction at this filling gives rise to a variety of sine-Gordon like terms in the bosonization treatment [38,40,41]. A Luttinger liquid calculation for carbon nanotubes then shows that for $\Delta \ll \hbar\omega \ll k_B T$, such terms may be treated as a perturbation and give rise to a correction to the conductivity which goes as $\delta\sigma(\omega) \sim \Delta^2 T^{1-2K}$ [3,40]. For $\eta \gg 1$, the integrated flux $\dot{N}_{\mathrm{dip}}$ would receive a correction scaling as $\sim T^{2-2K}$.

*Experimental considerations.* We now discuss the implications of our theory for carbon nanotubes. A typical value of the plasmon velocity is $v = 3.0 \times 10^6$ m/sec, with $K \approx 0.3$. At $T = 800$ K, with $\tau_0 = 1$ psec, we have $\eta \approx 100$ [42–44]. Integrating Eq. (10) gives $\dot{N}_{\mathrm{dip}}/L \approx 2 \times 10^6$ photons/(sec $\mu$m). The scaling relation $\dot{N} \propto L$ is expected to hold as long as the length of the tube $L$ satisfies $L \gg v\tau_0 \approx 1\,\mu$m. The radiated power is given by $4 \times 10^5$ eV/(sec $\mu$m). For radiation in the quadrupole channel, $\dot{N}_{\mathrm{quad}} \approx 0.3(v/c)^2\dot{N}_{\mathrm{dip}} \approx 10$ photons/(sec $\mu$m). For a given experimental setup, the total absorbed power will depend on the solid angle subtended by the detector.

Measurement of the quadrupole contribution to the radiation requires first establishing the symmetry axis of the emitter. Then, a comparison of the radiation emitted (either the number of photons or the power per unit time) at two distinct angles $\theta_1$ and $\theta_2$ can be used to fix the ratio of radiation in the dipole and quadrupole channels, allowing a determination of $v$ and $K$ provided $v_F$ is known. In addition to the 'intrinsic' quadrupole moment predicted here, an additional contribution to the quadrupole radiation would arise for a finite length system even if the radiation were emitted only in the dipole channel. This geometric effect would scale as $L^2$ whereas the quadrupole effects described here are proportional $L$.

## IV. CONCLUSIONS

In this work, we have developed a general theory of radiation from a 1D non-Fermi electron liquid. We find that dipole and quadrupole radiation taken together can be used to pinpoint Luttinger behavior in a 1D system. An interesting aspect of our results is that in the strongly damped limit, the dipole spectrum of a LL coupled to an Ohmic bath mimics the spectrum of a three-dimensional black body which may partially explain a previous experiment [29]. A clear signature of TLL physics is the characteristic power-law renormalization we predict due to Mott insulating behavior. Quadrupole radiation also bears subtle signatures of nonlinear TLL effects by revealing the interactions between bosonic quasiparticles. Our work is readily generalized to experimental setups that measure optical conductivity and demonstrates that rich physics is encoded in the quadrupole radiation profile such as the Luttinger liquid parameter. More broadly, integrating strongly-correlated electron systems with nanophotonic devices could allow for more novel probes of the radiative output.

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