An "optical Möbius strip." [Image: Dan Curticapean, University of Applied Sciences Offenburg, Germany; 1st Place, OPN 2011 Photo Contest]

Topological Photonic Systems

Sunil Mittal, Wade DeGottardi and Mohammad Hafezi

Opportunities are growing for the design of photonic systems that are "topologically protected" from defects and disorders—and in which light can be made to propagate in a single preferred direction.

Photonic crystals are materials whose refractive index varies in a periodic way. And, just as with electronic bands in an atomic crystal, electromagnetic waves can propagate in a photonic crystal in certain allowed frequency ranges, or bands, and are forbidden to propagate in other frequency ranges, or band gaps. Whether photonic or electronic, the usual technique to characterize these bands and band gaps has been to study the dispersion relation—that is, how the allowed frequency (energy) changes as a function of wavevector.

In the 1980s, however, a number of researchers realized that, in addition to these local properties, band structures possess global properties associated with topological features—that is, features related to the underlying symmetry of the material. The theory led, in condensed-matter physics, to the discovery of so-called topological insulators, materials that are conducting on their edges but insulating in their interiors. It also led to the 2016 Nobel Prize in Physics for three of its pioneers, David Thouless, and Duncan Haldane and J. Michael Kosterlitz.

Since the beginning of the current decade, implementations of topological photonic systems have likewise begun to yield rich results. In particular, systems that include so-called topological phase boundaries exhibit unusual effects, known as edge states, that allow unidirectional propagation of photons—even around corners—despite physical imperfections in the medium. And experiments with topological photonic systems are beginning to point to new approaches to the design of lasers, optical components and sources of quantum light.

Topological phases and edge states

Broadly speaking, topology is a branch of mathematics involving geometric and spatial properties that remain unchanged under continuous deformation. In the context of photonics, a system's topology is mathematically defined by variations in the eigenvectors of the system's Hamiltonian, $H(\mathbf{k})$, across all allowed values of the wavevector $\mathbf{k}$ (see infographic below).

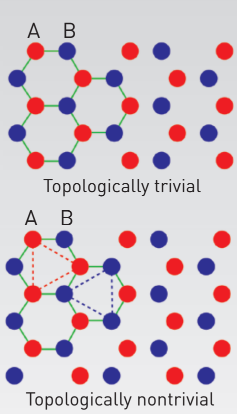
A simple example of topological states can be seen in a photonic system involving dielectric rods (or ring resonators) arranged in a 2-D honeycomb pattern—an analog of the so-called Haldane model of a topological insulator in an electronic system. In the photonic lattice, the unit cell involves two sites (A and B in the infographic). The photonic field is concentrated on the dielectric rods due to their high refractive index, and as a result of the evanescent field coupling, photons can "hop" (or leak) from one site to another.

In this system, the Hamiltonian $H(\mathbf{k})$ is a 2×2 matrix that describes the hopping between the A and B sublattices, with eigenfunctions representing the field strength and relative phase of photons on the sublattices. In a *topologically trivial* phase, only hops between nearest neighbors (A to B or B to A) are allowed. *Topologically nontrival* behavior can arise from a band gap induced by hopping among next-nearest neighbors (A to A and B to B). These A and B sublattices can also correspond to two different light polarizations in a photonic crystal.

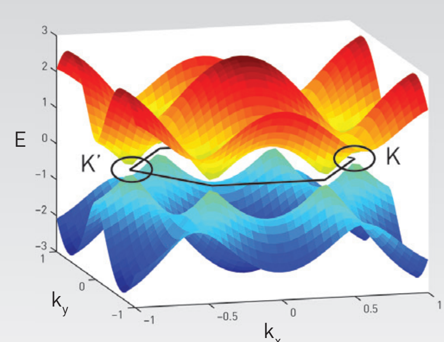
The topology of the band structure depends sensitively on the magnitudes and phases of the parameters in $H(\mathbf{k})$, and is a subtle effect. In particular, it can't be determined simply by looking at the dispersion properties of the medium. Instead, determining the topology requires investigating how the photonic states behave globally for relevant values of $\mathbf{k}$.

Physically, the implications of topological states emerge when we consider the case where a topologically trivial lattice (in the honeycomb example, one that permits hopping only among nearest neighbors) interfaces with a nontrivial lattice (one that allows hopping among next-nearest neighbors)—a topological phase boundary. Both lattices have similar energy bands, but their field distributions are very different. It is thus impossible to go smoothly from one lattice to the other. Further,

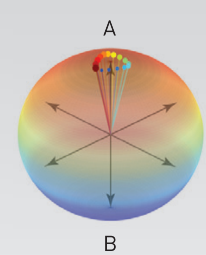
Describing topological order: The Haldane model



1. One model to illustrate a topological band structure is based on a honeycomb lattice, in which photons can hop between nearest neighbors (solid lines) or next-nearest neighbors (dashed lines) because of evanescent field coupling.



2. The dynamics of the photonic system are described by the Hamiltonian $H(\mathbf{k})$, a matrix characterized by the wavevector $\mathbf{k}$. The photonic band structure (shown above for the honeycomb model) corresponds to the frequency eigenvalues, $\omega(\mathbf{k})$, of this matrix.



3. The eigenvectors of $H(\mathbf{k})$ are two-component vectors representing the field strength and relative phase of photons on the A and B sublattices. They can be represented on the surface of a Bloch sphere, on which the latitude represents the relative field strength on the A and B sublattices. At the poles, the photonic field is entirely concentrated on a single site (A or B); at the equator, the two field amplitudes are equal.

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“Topological protection” against defects holds the key to the usefulness of topological properties in designing photonic components and systems.

while either side of the interface has a band gap, the band gap disappears at the interface—which implies that photons can travel *along* that interface.

These interface states, or edge states, are characteristic of topological systems. More important, it turns out that the edge states are very robust against disorder. For instance, in the honeycomb example, the ability for photons to “hop around” a defect means that a missing dielectric rod cannot change the photon propagation direction. This “topological protection” against defects holds the key to the usefulness of topological properties in designing photonic components and systems.

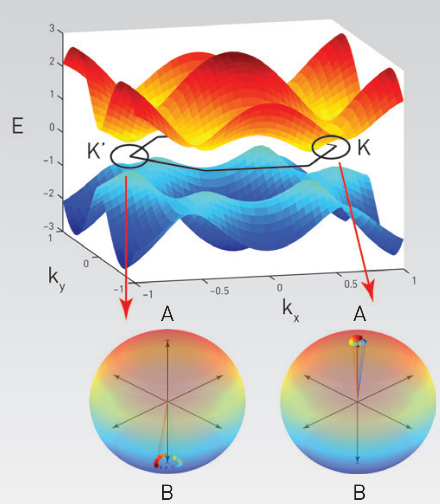
Creating topological photonic platforms

The example above gives an idea of what a topological band structure means, and how it gives rise to protected edge states. But under which physical conditions can such topological features be realized?

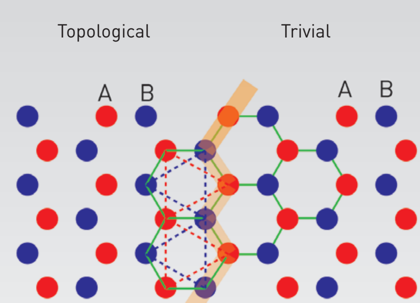
To understand this, consider another key example of an electronic topological system: the integer quantum Hall effect (IQHE). In a system in which electrons are confined on a 2-D plane, with a uniform magnetic field oriented perpendicular to the plane (a common scenario in semiconductor heterostructures), electrons within the plane will experience a cyclotron motion due to the Lorentz forces exerted by the magnetic field (see infographic below).

The net charge carried by electrons thus vanishes in the plane’s bulk, because cyclotron electrons are confined to circular orbits and don’t contribute to the current. At the plane edges, however, the electrons cannot complete their orbits and bounce off the edge. The boundary between the topologically nontrivial plane and the topologically trivial vacuum surrounding it thus carries current owing to these skipping orbits—and, indeed, carries current in opposite directions at opposite edges of the sample, allowing current to propagate unidirectionally around the plane edges.

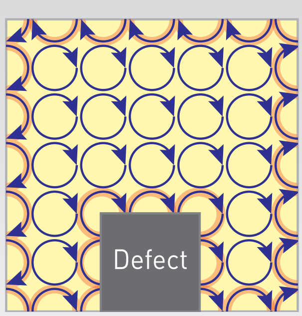
In the integer quantum Hall effect, it is the magnetic field—a gauge field—that gives rise to topological effects.



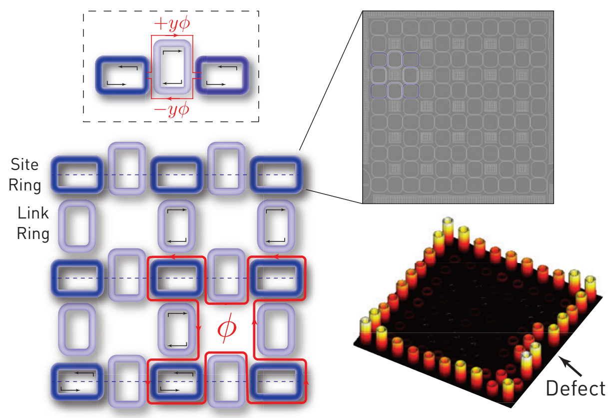
4. The system’s topology is described by the eigenvectors at the special symmetry points K and K’ in the Brillouin zone. If only the nearest-neighbor hops are allowed, the eigenfunctions associated with K and K’ visit the same pole of the Bloch sphere and the system is topologically trivial. If the next-nearest-neighbor hops are allowed as well, the field distribution is very different; the eigenfunctions visit different poles and the system is topologically nontrivial.



5. Topologically protected edge states arise at the phase boundary between topologically trivial and topologically nontrivial regions.



6. In the integer quantum Hall effect, edge states between the plane and the vacuum surrounding it allow current to flow unidirectionally around the plane edge (see text).



Coupled ring resonators

In a coupled-ring-resonator system (left and top right), asymmetric placement of resonators leads to differences in the accumulated phases for photons travelling around a path in the counterclockwise versus clockwise direction. The result is a topologically protected edge state, in which photons can propagate even around corners or around a defect (bottom right).

M. Hafezi et al. Nat. Photon. **7**, 1001 (2013).

Specifically, electrons circulating in a loop acquire a so-called Aharanov-Bohm phase that is direction-dependent (that is, reversing the direction of circulation reverses the sign of the phase). Photons are uncharged, and thus it's not evident how to achieve a topological photonic system if it requires an Aharanov-Bohm phase. In the last decade, researchers have come up with a variety of ingenious schemes to engineer such phases for photons.

Gyrotropic medium

Even though photons in vacuum do not respond to a magnetic field, in a gyromagnetic medium a magnetic field can alter the refractive index and, hence, the propagation of photons via the magneto-optic or Faraday effect. In 2008, Duncan Haldane and Srinivas Raghu proposed using this effect to implement topological edge states for photons, a proposal soon realized experimentally with microwaves. But using this scheme to realize topological edge states in nanophotonic and integrated-photonic systems is impractical, as magneto-optic effects are very weak at optical frequencies, such as those used in telecommunications.

Faking a magnetic field: Coupled ring resonators

A more rewarding avenue for implementing topological photonic systems is to engineer a synthetic magnetic field for photons. One such topological photonic platform uses a two-dimensional square lattice of ring resonators positioned at the lattice sites, coupled by "link" rings that allow a photon to hop from one lattice site to its neighbors via the link rings. To introduce a synthetic magnetic field in this system, the link rings are positioned asymmetrically between the site rings, so that a photon hopping along the positive x axis travels a slightly longer path when compared to a photon hopping along the negative x axis.

The difference in path length means that the photons acquire a direction-dependent hopping phase, and the link ring positioning guarantees that the synthetic magnetic flux is uniform throughout the lattice. This arrangement is similar to the IQHE, where electrons are confined to a 2-D plane under the influence of an external magnetic field. An experimental realization (fabricated using CMOS-compatible silicon-on-insulator technology) showed that edge states are confined to the rings on the boundary of the lattice, as expected.

Quantum spin Hall effect

Unlike a real magnetic field, the synthetic field in the example above does not break time-reversal symmetry. As a result, there are actually two instances of the IQHE associated with two pseudo-spins that describe photons in the ring resonators travelling clockwise and counterclockwise. This system is thus formally analogous to the quantum-spin Hall effect (QSHE), in which electron spin couples with orbital angular momentum and results in a counter-propagating edge state associated with each spin. Edge states are robust as long as the two spins remain uncoupled. (In the experimental coupled-resonator system, the use of directional couplers ensured that such spin-flip processes were negligible.)

Another example of a topological system analogous to the QSHE for photons emerges in the honeycomb lattice of dielectric rods described earlier. Here, the transverse magnetic-field solutions of the lattice correspond to fields with positive and negative angular momenta that are right and left circularly polarized in the plane of the lattice, and that act as the two pseudo-spins. Expanding the spacing between the dielectric rods can change the behavior of the wave function sufficiently to create a nontrivial topological phase and band gap, with edge states appearing the interface between the shrunken and the expanded lattices. At the interface, the two pseudo-spins travel in opposite directions, and as long as they remain uncoupled, they are robust against disorder.

The unidirectional propagation of topological edge states and their robustness against backscattering is particularly appealing for photonic integrated circuits (PICs).

Floquet systems

Time modulation, like a magnetic field, can explicitly break time-reversal symmetry, and thus give rise to a topologically nontrivial phase. For example, consider a different coupled-resonator system, in which ring resonances at alternate sites differ by a frequency Ω, such that the neighboring sites are effectively uncoupled. Then a modulation of each site, also at frequency Ω, leads to a coupling between the neighboring sites. By controlling the phase of the modulation at different sites, it is possible to introduce a synthetic magnetic field for photons. Such time-periodic topological systems are associated with periodic energy bands, and are known as Floquet topological systems.

Interestingly, the Schrödinger equation governing the temporal evolution of a wave packet is very similar to the spatial propagation of a light beam in the paraxial approximation. This implies that the temporal modulation of the 2-D lattice can be simulated using spatial modulation—that is, a 2-D lattice of helical waveguides. Here, propagation of light along the waveguides simulates evolution of time, and the helicity of the waveguides simulates modulation. Such a 2-D array of helical waveguides can be inscribed easily in fused silica, using femtosecond laser writing.

Exciton-polaritons

Unlike electrons, photons do not interact with each other, and the nonlinear interactions mediated by the medium's refractive index are also very weak. This severely limits the kind of models of topological systems that can be simulated using photons. An alternative is to use exciton-polaritons—light-matter quasiparticles in a semiconductor cavity, in which electronic excitations in the semiconductor material couple with photonic excitations in the cavity. By coupling many such cavities in 2-D hexagonal arrays, it is possible to realize topological edge states.

Some potential applications

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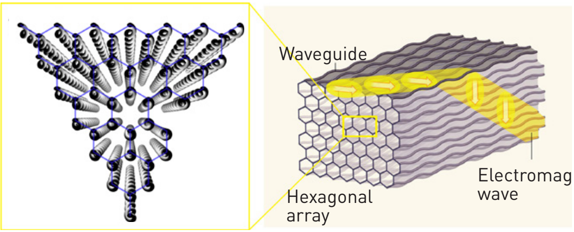


A 2-D lattice of helical waveguides in which z-symmetry is broken simulates, in space, the breaking of time-reversal symmetry characteristic of Floquet topological insulators.

M.C. Rechtsman et al. Nature **496**, 196 (2013) / Y. Chong, Nature **496**, 173 (2013); reprinted by permission of Nature, ©2013

A number of recent demonstrations have pointed to the application potential of photonic topological edge states.

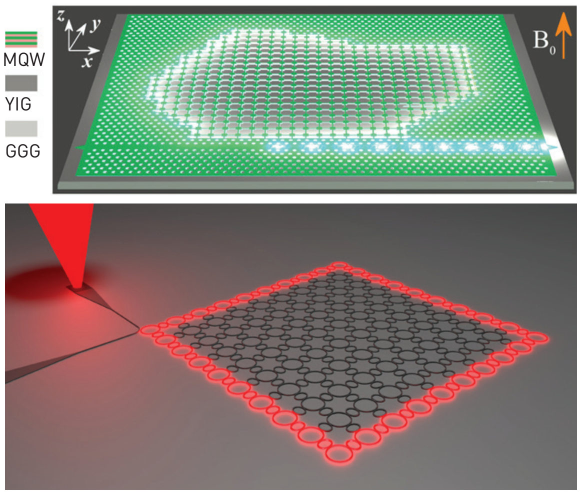
Robust optical delay lines

In on-chip, all-optical information processing, optical delay lines are essential for storing photons for a finite time to compensate for processing delays. Simple waveguide spirals can introduce delays of hundreds of picoseconds, but they have a large footprint. Coupled resonator arrays, or so-called slow-light waveguides, can reduce the effective group velocity of photons significantly because photons undergo multiple round trips in each resonator. However, fabrication disorder leads to variations in the ring resonance frequencies that can exponentially decrease or even completely halt the forward transmission of light.

Because topological edge states are essentially quasi-1-D channels confined to the lattice boundary, the edge states of a coupled-resonator system can effectively work as delay lines, while remaining robust against disorder. A comparison of edge-state transport with similarly designed topologically trivial 1-D resonator arrays revealed that in the presence of disorder, edge states did indeed achieve higher transmission than the trivial arrays, pointing to the topological system's potential use in optical delay lines in PICs.

Topological lasers

Very recently, the concept of topological protection has been extended to the design of lasing cavities using



Topological lasers

Top: A topological laser with an arbitrarily shaped cavity, designed using gyromagnetic medium (yttrium iron garnet, YIG), grown on gadolinium gallium garnet (GGG) and overlain by an active layer of InGaAsP multiple quantum wells (MQW). Bottom: A topological laser using a coupled-ring-resonator architecture. In both cases, only the edge modes achieve lasing threshold.

B. Bahari et al., Science **358**, 636 (2017) / M.A. Banders et al. Science, doi: 10.1126/science.aar4005 (2018); reprinted with permission from AAAS

material platforms with optical gain. The lasing threshold of a laser cavity is very sensitive to fabrication disorder and cavity shape—bends in the cavity, for example, can introduce loss and increase the lasing threshold. Topological edge states hold the prospect of routing around sharp bends without scattering loss, and thus of designing robust laser cavities with arbitrary shapes.

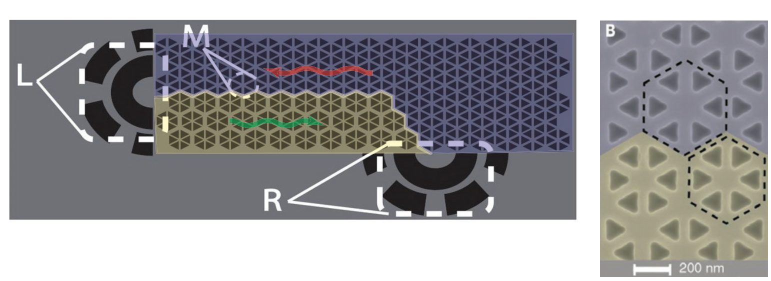
One such implementation, from researchers at the University of California, San Diego, USA, used a real magnetic field with a gyromagnetic medium, cleverly segregating the functionalities of topological protection and the optical gain. A photonic-crystal layer, fabricated in the gyrotropic medium yttrium iron garnate (YIG), allows the application of a magnetic field to open up a small topological band gap. Another layer, consisting of InGaAsP multiple-quantum-well (MQW) structures, is bonded on top of the YIG layer, and pumped with a 1064-nm beam to induce optical gain.

The laser cavity in this design lies in the edge states localized at the interface between the topologically trivial and nontrivial regions, and is therefore robust against deformations of the cavity shape. While this design can work at telecom wavelengths and at room temperature, one downside is that the use of YIG and a magnetic field is not compatible with CMOS fabrication processes. Further, the lasing is confined to a very narrow spectrum of around 50 pm, where the magneto-optical effects are significant.

In another approach, researchers at The Technion, Israel, and the University of Central Florida (CREOL), USA, used a coupled-ring-resonator system, fabricated using a gain medium, to realize topological lasers. The system uses asymmetric positioning of link rings to create a synthetic magnetic field for photons and implement the IQHE. As with the previous example, the ring resonators are fabricated using InGaAsP and pumped with a 1064-nm laser. A specially designed metal mask on top of the resonators ensures that the pump beam illuminates only the resonators on the lattice edges, and thus only the edge modes of the system achieve lasing threshold. In contrast, because of the absence of a pump beam in the bulk material, bulk band wave functions experience an additional loss and are therefore not amplified.

Topological quantum photonic systems

Quantum systems are even more fragile and susceptible to disorder than classical systems, so the concept of topological robustness and photonic edge states is especially intriguing in the context of integrated quantum photonic devices for applications in quantum information processing, sensing and metrology. Researchers have recently begun using topological properties to demonstrate a robust source of quantum light.



Topological quantum emitter

A photonic crystal, with different hole spacings in the yellow and blue areas, created a robust edge state that transported single photons emitted by a quantum dot excited at point M, even in the presence of a bend in the waveguide. Emissions were collected at gratings at positions L and R.

Quantum systems are even more fragile and susceptible to disorder than classical systems, so the concept of topological robustness and photonic edge states is especially intriguing.

Quantum light in general constitutes single photons, heralded photons and correlated photon pairs. The most commonly used techniques to generate these photons rely on processes such as spontaneous parametric down-conversion and spontaneous four-wave mixing in nonlinear media. The spectrum of the generated photons is governed by energy and momentum conservation and by the nonlinear material's underlying dispersion relations or, equivalently, the density of electromagnetic modes.

Nanophotonic systems are a convenient platform for engineering the electromagnetic mode structure and the spectrum of generated photons. But fabrication disorders can severely and randomly alter the electromagnetic mode structure, resulting in an unpredictable spectrum for the generated photons. This is where the topological protection comes in handy—in a topological band, the density of modes and, therefore, the dispersion relation is robust against disorder. By exploiting this robustness in a coupled-resonator system, researchers have demonstrated a topological source of quantum light in which topological protection yields a robustness in the spectrum of generated photons.

Another recent case, from our lab, demonstrated a topological quantum optics interface between a quantum dot and a photonic crystal that realizes the QSHE. The photonic crystal design in this scheme is very similar to honeycomb array of rods described earlier, except that, instead of rods, the photonic crystal consists of holes etched in suspended dielectric membrane. This structure exhibits a pair of counter-propagating edge states at the interface between two topologically distinct regions with expanded and shrunken cells.

The two counter-propagating edge states are associated with two counter-rotating circular polarizations of the electric field—that is, the left-circularly-polarized field can only travel to the right, and vice versa. Instead of classical sources, quantum dots were used to excite the edge states. The use of quantum dots means that an applied magnetic field results in Zeeman splitting of the excited level into two spin-polarized levels, one coupled to the right circular polarization and the other to the left circular polarization. This spin polarization, in turn, was subsequently used to excite the left- and right-propagating edge states at the interface.

Future prospects

Although a number of platforms have demonstrated proof-of-concept, robust topological photonic systems, only a few concrete demonstrations have shown the advantages of topological robustness over trivial systems. Future research will focus on expanding these demonstrations into useful applications, by comparing topologically protected devices with trivial ones—for example, 2-D topological systems with 1-D waveguide systems using the same design parameters, fabrication process and material properties.

Moreover, while researchers have shown the robustness of topological states in circumventing sharp bends and defects, the robustness of the underlying mode structure still has not been fully utilized. We see an interesting future in using topological engineering of electromagnetic modes for manipulating, at a very fundamental level, light-matter interactions, and also many-body interactions in quantum photonic systems. OPN

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