

Candidate for a Passively Protected Quantum Memory in Two Dimensions

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An interesting problem in the field of quantum error correction involves finding a physical system that hosts a “passively protected quantum memory,” defined as an encoded qubit coupled to an environment that naturally wants to correct errors. To date, a quantum memory stable against finite-temperature effects is known only in four spatial dimensions or higher. Here, we take a different approach to realize a stable quantum memory by relying on a driven-dissipative environment. We propose a new model, the photonic-Ising model, which appears to passively correct against both bit-flip and phase-flip errors in two dimensions: a square lattice composed of photonic “cat qubits” coupled via dissipative terms which tend to fix errors locally. Inspired by the presence of two distinct \mathbb{Z}_2 -symmetry-broken phases, our scheme relies on Ising-like dissipators to protect against bit flips and on a driven-dissipative photonic environment to protect against phase flips. We also discuss possible ways to realize the photonic-Ising model.

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Quantum error correction remains one of the biggest challenges toward building a practical quantum computer [1,2]. One of the leading candidates for realizing fault tolerance is the family of quantum stabilizer codes [3], including the surface code [4–6] and the Gottesman-Kitaev-Preskill code [7]. These error-correcting schemes are based on fast error recovery controlled by the feedback from repetitive syndrome measurements.

A prominent alternative is the finite-temperature quantum memory: Certain thermal environments naturally evolve arbitrary initial states into a qubit subspace of interest at low temperature, thus eliminating the need for active measurements and correcting operations. Many recent studies have investigated thermal self-correcting properties [6,8–20]. To date, the only known models that host a passive quantum memory via this mechanism are topological codes in four dimensions (4D) and higher, e.g., the 4D toric code [6,19].

A separate line of research aims to uncover a passively protected quantum memory via engineered “driven-dissipative” systems [21–41]. Such passive protection includes but is not limited to the finite-temperature case, since a thermal-equilibrium steady state is not required. The memory is dynamically protected against certain noise channels by (local) Markovian dissipation. This has led to a number of new ideas for passive error correction, such as the autonomously corrected cat qubit [42,43] and the dissipative Toom’s rule [6,28,44]. Unfortunately, none of these models can protect a quantum memory for an exponentially long time as a function of the system size (in less than four dimensions).

In this work, we study a model with engineered dissipation which appears to protect against both bit flips and phase flips and lives in two spatial dimensions. Instead of relying on topological order, we suggest that the model should belong to a phase that spontaneously breaks two different \mathbb{Z}_2 symmetries. Each \mathbb{Z}_2 -symmetry-broken phase protects a “classical bit,” which together form a robust qubit. The proposed model provides an example of a robust quantum memory which, at low temperature, can be exponentially long-lived in system size parameters, albeit with challenging physical requirements.

Quantum memory.—Consider a Hilbert space \mathcal{H} , and define two encoded, logical states $|\bar{0}\rangle, |\bar{1}\rangle \in \mathcal{H}$ that span the code space \mathcal{C} . We assume the system is always initialized in the code space: $\rho_i = |\psi\rangle\langle\psi|$, where $|\psi\rangle \in \mathcal{C}$.

A local continuous-time Markovian generator \mathcal{L} in Lindblad form is defined by

$$\frac{d\rho}{dt} = \mathcal{L}(\rho) = -i[H, \rho] + \sum_j \left(L_j \rho L_j^\dagger - \frac{1}{2} \{L_j^\dagger L_j, \rho\} \right), \quad (1)$$

where H is the Hamiltonian of the system and L_j are local dissipators which arise due to the system-environment coupling [45]. We consider a dynamical process that can be decomposed into two parts, an “error” generator and a “recovery” generator: $\mathcal{L} = \mathcal{L}_e + \mathcal{L}_r$. The error generator describes the main channels of physical noise which move the initial state out of the code space. The recovery generator stabilizes the code space: $\mathcal{L}_r(\rho_i) = 0$; i.e., any state in the code space is a steady state of the recovery. We

allow for this noisy process to occur for a time t , which generically sends ρ_i to a mixed state $\rho_m(t) = e^{\mathcal{L}t}(\rho_i)$.

Finally, we employ a ‘‘single-shot’’ decoding quantum channel \mathcal{E}_r which sends every state in the Hilbert space back to the code space [46]. The final state is

$$\rho_f(t) = \mathcal{E}_r e^{\mathcal{L}t}(\rho_i). \quad (2)$$

We wish to find systems where the difference between the initial and final states is exponentially small in the system size:

$$1 - \text{Tr}[\rho_i \rho_f(t)] = O(e^{-\gamma M}) \quad \text{as } M \rightarrow \infty, \quad (3)$$

where $\gamma > 0$ is a time-independent constant and M is some system size parameter. A system described by \mathcal{L} hosts a passively protected quantum memory for any finite time t if Eq. (3) holds as the thermodynamic limit is approached.

The bit-flip and phase-flip errors of a two-level system are generated via the Pauli operators X and Z , respectively. A good quantum memory should, thus, protect against both sources of noise. Recent work [20,40] has described the connection between \mathbb{Z}_2 symmetry breaking and error correction: A symmetry-broken phase protects quantum information against X or Z errors but not both. This leads to a protected classical bit, which can be viewed as a quantum bit experiencing biased noise [47].

In this work, we attempt to glue two different classical bits together to form a robust qubit. Our strategy involves studying a system that passively corrects against bit flips due to Ising-like dissipators which tend to align qubits locally. Furthermore, phase flips will passively correct due to driven-dissipative stabilization of the photonic cat code. We begin by describing spontaneous symmetry breaking in the cat code and in the Ising model separately. We then describe a model which inherits both protecting features.

Photonic cat code.—Let us briefly review \mathbb{Z}_2 spontaneous symmetry breaking in the photonic cat code [29,48]. (For a detailed analysis, we refer to Ref. [40].) Consider a driven-dissipative photonic cavity in the presence of two-photon drive and two-photon loss. The rotating-frame Hamiltonian and dissipator read $H = \lambda(a^2 + (a^\dagger)^2)$ and $L_2 = \sqrt{\kappa_2}a^2$, respectively. Here, a is the annihilation operator for a cavity photon, λ is the drive strength, and κ_2 is the two-photon loss rate. While the model has \mathbb{Z}_2 symmetry $[H, Q] = [L_2, Q] = 0$ generated by parity $Q = e^{i\pi a^\dagger a}$, the steady state can violate this symmetry:

$$\rho_{\text{ss}} = |\psi\rangle\langle\psi|, \quad |\psi\rangle = c_0|\alpha_e\rangle + c_1|\alpha_o\rangle, \quad (4)$$

for $|c_0|^2 + |c_1|^2 = 1$, where $|\alpha_e\rangle \sim |\alpha\rangle + |-\alpha\rangle$, $|\alpha_o\rangle \sim |\alpha\rangle - |-\alpha\rangle$, and $|\alpha\rangle$ is a coherent state with amplitude $\alpha = e^{-i\pi/4}\sqrt{N}$ and $N \equiv \lambda/\kappa_2$ photons. The even and odd cat states $|\alpha_{e/o}\rangle$ represent logical 0 and 1, respectively.

The cat code is protected against phase-flip errors generated by photon dephasing $L_d = \sqrt{\kappa_d}a^\dagger a$. Indeed, the phase-flip logical error rate scales as $e^{-\gamma N}$, where γ is a constant [29]. The symmetry-broken states $|\pm\alpha\rangle \approx (|\alpha_e\rangle \pm |\alpha_o\rangle)/\sqrt{2}$ have an exponentially long lifetime in the limit of large N , ensuring that logical phase flips are unlikely.

The dominant decoherence mechanism for the cat qubit stems from the bit flip, generated via single-photon loss $L_1 = \sqrt{\kappa_1}a$: $a|\alpha_{e/o}\rangle \sim |\alpha_{o/e}\rangle$, which reduces the qubit steady state structure to a classical bit: $\rho_{\text{ss}} \approx c|\alpha\rangle\langle\alpha| + (1-c)|-\alpha\rangle\langle-\alpha|$, $c \in [0, 1]$ [40]. More generally, perturbations that commute with photon parity (e.g., $[L_d, Q] = 0$) are expected to be passively corrected, while terms which explicitly break the symmetry (e.g., $\{L_1, Q\} = 0$) are not.

2D Ising model.—We now turn our attention to a system that has the opposite problem: \mathbb{Z}_2 symmetry breaking will protect against bit flips but not phase flips. We consider the 2D Ising model on an $M \times M$ square lattice with periodic boundary conditions. The Hamiltonian reads

$$H_{\text{Is}} = - \sum_{x,y=1}^M (Z_{x,y}Z_{x+1,y} + Z_{x,y}Z_{x,y+1}), \quad (5)$$

where $Z_{x,y}$ is the Z Pauli operator on site (x, y) . The ferromagnetic states are the ground states of this model and span the code space: $|\bar{0}\rangle \equiv |\downarrow\downarrow\downarrow\dots\rangle$, $|\bar{1}\rangle \equiv |\uparrow\uparrow\uparrow\dots\rangle$, with $Z|\downarrow\rangle = |\downarrow\rangle$ and $Z|\uparrow\rangle = -|\uparrow\rangle$.

We define local dissipators that describe the thermalization of the Ising Hamiltonian. (For simplicity, we set the Hamiltonian in the master equation to zero.) Consider dissipators that are a product of a spin flip (X) with a projector onto a particular domain-wall configuration. These jumps will cause a spin to flip sign according to a local ‘‘majority rule,’’ i.e., only if more than two of the neighboring spins are misaligned. Specifically,

$$\begin{aligned} L_{x,y}^{(4)} &= \sqrt{\tilde{\kappa}} X_{x,y} P_{x,y;\rightarrow}^- P_{x,y;\uparrow}^- P_{x-1,y;\rightarrow}^- P_{x,y-1;\uparrow}^-, \\ L_{x,y}^{(3)} &= \sqrt{\tilde{\kappa}} X_{x,y} P_{x,y;\rightarrow}^+ P_{x,y;\uparrow}^- P_{x-1,y;\rightarrow}^- P_{x,y-1;\uparrow}^-, \end{aligned} \quad (6)$$

where $\tilde{\kappa} = \sqrt{\Delta\kappa + \Delta^2} - \Delta$ and $P_{x,y;\rightarrow}^\pm = (1 \pm Z_{x,y}Z_{x+1,y})/2$ and $P_{x,y;\uparrow}^\pm = (1 \pm Z_{x,y}Z_{x,y+1})/2$ are projectors onto particular local configurations of spins. The superscripts indicate the number of domain walls which the projector is checking for, and we neglect to write jumps related by rotational invariance (i.e., there are four different $L^{(3)}$ operators per site) [49]. We also consider an error process in the form of a uniform bit flip rate on each lattice site: $L'_{x,y} = \sqrt{\Delta} X_{x,y}$.

We have chosen our dissipators above such that the steady state of the model is the thermal state of the 2D classical Ising model:

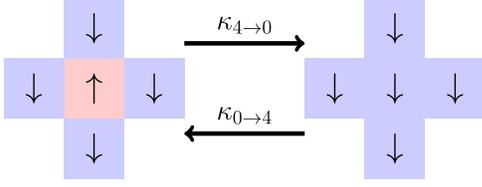


FIG. 1. The total rate of transitioning from a configuration with four domain walls to a configuration with zero domain walls satisfies detailed balance: $\kappa_{4\rightarrow 0}/\kappa_{0\rightarrow 4} = e^{8\beta}$.

$$\rho_{\text{ss}} = \frac{e^{-\beta H_{\text{Is}}}}{\text{Tr}[e^{-\beta H_{\text{Is}}}]}, \quad \beta = \frac{1}{8} \ln \left[\frac{\kappa + \Delta}{\Delta} \right], \quad (7)$$

with the effective temperature set by the relative ratio of the correction rate to the bit-flip rate. Within the quantum jump picture [50,51], the rates of transitioning between different classical configurations respect detailed balance. (See, e.g., Fig. 1.)

While the thermal state (7) is always a steady state of the model, it is not unique. All dissipators commute with the parity operator $Q = \prod_{i=1}^{M^2} X_i$; $[L_j, Q] = 0$. This means that the dynamics preserves the parity of the state (called a “strong \mathbb{Z}_2 symmetry” [52]). In the thermodynamic limit of the low-temperature (symmetry-broken) phase, a qubit can be stored in the steady state [40].

We can confirm this picture via numerical simulations. Suppose we initialize our system in a ferromagnetic state: $|\psi\rangle = |\bar{0}\rangle = (|E_0^+\rangle + |E_0^-\rangle)/\sqrt{2}$, where $|E_0^\pm\rangle$ are ground states in the different parity sectors [53]. We then quench the system with the noisy Lindbladian for a time T much larger than the inverse of the dissipative gap, so that the system settles into its steady state. Finally, we apply a single-shot decoder which brings the state back to the code space by measuring all domain walls in the system then flipping all bits in the smaller domain. Our results are summarized in Fig. 2. In the low-temperature phase, the overlap starts to approach the ideal value of 1 exponentially fast in M . Qualitatively different behavior occurs in the high-temperature phase [$\beta > \beta_c = \ln(1 + \sqrt{2})/2 \approx 0.44$; red dots], where the success rate stays at 50% for a wide range of M .

Unfortunately, the stored qubit is unstable to noise that violates the strong symmetry. In particular, the presence of dephasing $L_i \sim Z_i$ (phase flips) reduces the strong \mathbb{Z}_2 symmetry to a “weak \mathbb{Z}_2 symmetry” [defined at the level of the superoperator: $[\mathcal{L}, \mathcal{Q}] = 0$, where $\mathcal{Q}(\rho) = \mathcal{Q}\rho\mathcal{Q}^\dagger$], such that only a classical bit can be stored in the steady state. In this case, the steady state at low temperature has the structure $\rho_{\text{ss}} \approx c|\bar{0}\rangle\langle\bar{0}| + (1-c)|\bar{1}\rangle\langle\bar{1}|$, for $c \in [0, 1]$. In analogy with the cat qubit in the presence of single-photon loss, Z dephasing destroys the coherence between Ising ferromagnetic states.

2D photonic-Ising model.—We see that the cat code passively corrects against phase flips but not bit flips and

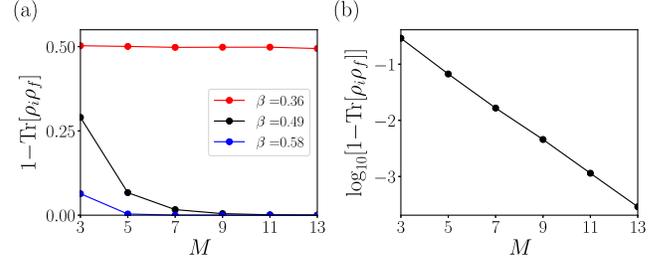


FIG. 2. (a) The overlap between the initial and final states for the protocol given in the main text, for a Lindbladian in the high-temperature phase (red dots) and in the low-temperature phase (black and blue dots). As linear system size M grows, the overlap approaches one only in the low-temperature (symmetry-broken) phase corresponding to $\beta > \beta_c \approx 0.44$. (b) The same black data points on a log plot; the overlap tends to one exponentially fast in M . In both (a) and (b), the quench time is $T = 800/\kappa$, i.e., long enough to reach the steady state. The simulation employs the quantum jump approach by averaging over 10^5 trajectories.

that the 2D Ising model passively corrects against bit flips but not phase flips. Is it possible to combine the protecting features of both models to construct a system that passively corrects against both sources of noise?

Consider an $M \times M$ square lattice of photonic cavities. Each cavity undergoes a two-photon drive process and a two-photon loss process:

$$H_{x,y} = \lambda(a_{x,y}^2 + (a_{x,y}^\dagger)^2), \quad L_{2,x,y} = \sqrt{\kappa_2} a_{x,y}^2, \quad (8)$$

where $a_{x,y}$ is the annihilation operator on site (x, y) . Next, let us consider a parity-parity interaction between neighboring cavities: $H_S = -\sum_{\langle ij \rangle} Q_i Q_j$, where Q_j is the photon parity operator at site j . Similar to the Ising model, at low temperatures, such interaction will tend to align the parities of neighboring cavities via the following local dissipators (for a microscopic derivation of the dissipators, see Supplemental Material [54]):

$$\begin{aligned} L_{x,y}^{(4)} &= \sqrt{\kappa_{nn}} a_{x,y} P_{x,y;\rightarrow}^- P_{x,y;\uparrow}^- P_{x-1,y;\rightarrow}^- P_{x,y-1;\uparrow}^-, \\ L_{x,y}^{(3)} &= \sqrt{\tilde{\kappa}_{nn}} a_{x,y} P_{x,y;\rightarrow}^+ P_{x,y;\uparrow}^- P_{x-1,y;\rightarrow}^- P_{x,y-1;\uparrow}^-, \end{aligned} \quad (9)$$

where $a_{x,y}$ is the annihilation operator for the cavity at site x, y , $\tilde{\kappa}_{nn} = \sqrt{\kappa_1 \kappa_{nn} + \kappa_1^2} - \kappa_1$, κ_1 is the single-photon loss rate (corresponding to the dissipator: $L_{1,x,y} = \sqrt{\kappa_1} a_{x,y}$), $P_{x,y;\rightarrow}^\pm = (1 \pm Q_{x,y} Q_{x+1,y})/2$, $P_{x,y;\uparrow}^\pm = (1 \pm Q_{x,y} Q_{x,y+1})/2$, and $Q_{x,y} = e^{i\pi a_{x,y}^\dagger a_{x,y}}$. The following states are the steady states of the model in the absence of errors ($\kappa_1 = 0$) and span the code space:

$$|\psi\rangle = c_0 |\alpha_e\rangle |\alpha_e\rangle |\alpha_e\rangle \dots + c_1 |\alpha_o\rangle |\alpha_o\rangle |\alpha_o\rangle \dots, \quad (10)$$

for $|c_0|^2 + |c_1|^2 = 1$.

For thermal systems, the existence of a passively correcting quantum memory is related to the presence of an extensive energy barrier which local errors must overcome in order to create a logical bit-flip or phase-flip operation [68]. In the model described above, a logical bit-flip operation can be created via local single-photon loss $L_{1,x,y} = \sqrt{\kappa_1}a_{x,y}$ only by passing through a configuration with an extensive number of domain walls, which is exponentially unlikely in the limit of large lattice size $M \rightarrow \infty$. Similarly, a phase-flip error can be generated only by taking the state $|\alpha_e\rangle \pm |\alpha_o\rangle$ to $|\alpha_e\rangle \mp |\alpha_o\rangle$ for any of the cavities. However, such a process is also unlikely to occur via dephasing perturbations $L_{d,x,y} = \sqrt{\kappa_d}a_{x,y}^\dagger a_{x,y}$ which perturb states locally in phase space, since the states $|\pm\alpha\rangle \approx |\alpha_e\rangle \pm |\alpha_o\rangle$ are well separated in phase space and an unstable fixed point sits between them [69]. The logical phase-flip errors are again exponentially unlikely as $N \rightarrow \infty$.

The single-photon loss and the dephasing lead to terms proportional to $a^\dagger a$ and $(a^\dagger a)^2$ in the Lindbladian, which result in leakage out of the effective two-level code space for each cavity into other states of the cavity. This leakage poses a challenge for numerical simulation, since (unlike the Ising model) we need to keep track of more than two degrees of freedom per lattice site. Nevertheless, we shall provide evidence for a stable quantum memory by employing a variety of approximations.

First, let us consider an approximation that allows us to map the dynamics of the photonic-Ising model directly to the classical-Ising model studied above. Specifically, we introduce an idealized model by replacing the single-photon loss dissipator $L_1 = \sqrt{\kappa_1}a$ with $E_1 = \sqrt{\kappa_1}b$, where $b = aV$ and V is the projector onto the code space: $V = |\alpha_e\rangle\langle\alpha_e| + |\alpha_o\rangle\langle\alpha_o|$. We also assume an absence of dephasing errors, i.e., $\kappa_d = 0$. This allows us to treat each site as an effective two-level system $|0\rangle = |\alpha_e\rangle$, $|1\rangle = |\alpha_o\rangle$, avoiding any leakage out of the code space. We similarly replace $a \rightarrow b$ in the nearest-neighbor coupling dissipators (9) (except in the definition of Q). The operator b can be regarded as an ‘‘idealized bit flip’’ since, for $N \gg 1$, it takes the form $b \approx \alpha(|\alpha_e\rangle\langle\alpha_o| + |\alpha_o\rangle\langle\alpha_e|)$. The idealized model maps exactly to the Ising model studied above, with an effective bit-flip error rate of $N\kappa_1$, an effective Ising-correction rate of $N\kappa_{nn}$, and an inverse temperature $\beta = \ln[(\kappa_{nn} + \kappa_1)/\kappa_1]/8$. We, therefore, find that this model passively corrects against bit flips in the limit $M \rightarrow \infty$ of the low-temperature phase. In the limit of large driving strength and small single-photon loss, we expect the photonic-Ising model to be well approximated by the idealized model, since the state rarely leaves the code space. We provide quantitative evidence for this in Supplemental Material [54].

Dephasing, single-photon loss, and bit-flip recovery jumps ($L_{x,y}^{(3)}$ and $L_{x,y}^{(4)}$) cause leakage out of the code space which is neglected within the idealized model. It is natural to ask whether this leakage is detrimental to the passively

protected memory when the idealized model is no longer a good approximation. We provide evidence that this is not the case by studying a toy model which resembles the 2D model. Consider a single cavity coupled to a spin-1/2 particle (described by Pauli operators X , Y , and Z), leading to two logical states $|\downarrow\rangle|\alpha_e\rangle$ and $|\downarrow\rangle|\alpha_o\rangle$. The Hamiltonian and jump operators read $h = \lambda(a^2 + (a^\dagger)^2)$, $l_2 = \sqrt{\kappa_2}a^2$, $l_1 = \sqrt{\kappa_1}Xa$, $l_d = \sqrt{\kappa_d}a^\dagger a$, and $l_{nn} = \sqrt{\kappa_{nn}}\frac{1}{2}X(1-Z)a$. The model assumes that single-photon loss is accompanied by a spin flip, while two-photon drive and dephasing are not. The flip-recovery jump l_{nn} is triggered by a flipped spin state $|\uparrow\rangle$, similar to the bit-flip recovery jump caused by a parity misalignment in 2D. Importantly, leakage caused by the noise processes l_1 and l_d , and the flip-recovery jump is captured by this model. In Supplemental Material [54], we analyze this model numerically and analytically. We find that the initial state can always be perfectly restored via a decoder (up to corrections exponentially small in N).

Finally, the stability of the memory can also be understood as the coexistence of two order parameters: $\langle Q \rangle = \langle e^{ina^\dagger a} \rangle \neq 0$ indicates the ferromagnetic phase and, therefore, suppression of bit-flip errors, while $\langle a^2 \rangle \neq 0$ indicates that the cat states are stabilized, implying suppression of phase-flip errors. We use a product-state mean-field ansatz $\rho = \otimes_{x,y=1}^M \rho_{x,y}$, where each $\rho_{x,y}$ is a density matrix for a two-level system in the basis of $|\pm\alpha_{\text{MF}}\rangle$ for some mean-field coherent parameter α_{MF} . A nontrivial dissipative phase of the system is identified by nonzero fixed points of $\langle Q \rangle$ and $\langle a^2 \rangle$. The mean-field solutions suggest that, for small κ_1 and κ_d , the memory is protected against both phase- and bit-flip errors. When κ_1 or κ_d exceeds a threshold, the order parameters undergo two second-order phase transitions and the quantum memory is no longer stable (see Supplemental Material [54]). The mean-field phase diagram is sketched in Fig. 3.

Implementing the photonic-Ising dissipators.—The key ingredients for our proposal are the microscopic dissipators defined in Eq. (9). A direct approach to achieve such terms involves engineering an Ising-like interaction between cavity modes: $H_S \propto -\sum_{\langle ij \rangle} Q_i Q_j$. The natural system-bath interaction of the form $\sum_i (a_i + a_i^\dagger) \otimes B_i$ (where B_i acts on bath degrees of freedom) would then give rise to the model described above (within the standard Born-Markov approximation) [54]. The parity-parity interaction H_S can be engineered from coupling between high-impedance cavity modes and Josephson junctions [70,71], as we review in Supplemental Material [54].

Inspired by the microscopic dissipators Eq. (9), an alternative approach to protect the memory involves digitally implementing a stochastic local error decoder. In Supplemental Material [54], we provide an explicit description of how to achieve such a local decoder autonomously without the need of measurements; however, it requires that

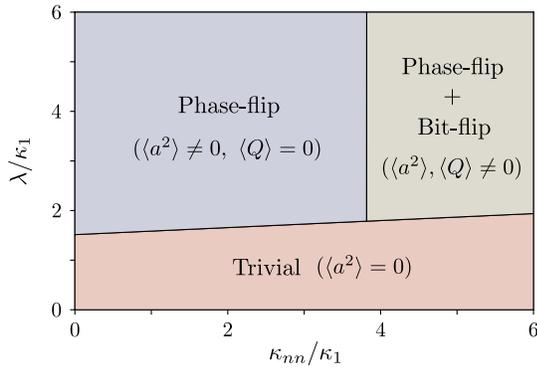


FIG. 3. The mean-field phase diagram for $\kappa_d = \kappa_1$. The top right corner shades the region where both $\langle Q \rangle$ and $\langle a^2 \rangle$ are nonzero. Both phase- and bit-flip errors are protected. When $\langle a^2 \rangle \neq 0$ but $\langle Q \rangle = 0$, we expect protection only for phase errors. When $\langle a^2 \rangle = 0$, we expect the memory to become fragile under either noise.

the local error decoding should be carried out at a rate that scales linearly in the photon number N of each cavity. The implementation can be achieved simultaneously with the dephasing protection Eq. (8), making it fault tolerant [69]. Note that this is different from the active repetition cat code in 1D [69] as we avoid the processing of nonlocal syndrome information.

Discussion and outlook.—We proposed a photonic-Ising model that hosts robust quantum memory under both single-photon loss and dephasing noise. We can estimate the logical error rates in the photonic-Ising model as follows. While the bit-flip error rate becomes extensive [$\sim O(N)$] in the limit of a large cavity photon number, the Ising-type interaction gives rise to an exponentially suppressed error rate $O(e^{-\gamma M})$ with $\gamma > 0$ [72–74], resulting in a logical bit-flip error rate of $O(Ne^{-\gamma M})$. Similarly, a single cavity yields a phase-flip error rate of $O(e^{-\gamma' N})$ with $\gamma' > 0$, while this is made extensive by the spatially extended lattice configuration, resulting in a logical phase-flip error rate of $O(M^2 e^{-\gamma' N})$. Harmonic oscillators with small nonlinearities and outstanding coherence properties—and, thus, with large achievable N —can be found in a variety of photonic and phononic systems (e.g., [39,75]).

The realization of the parity-parity coupling based on Josephson junctions and a high-impedance cavity mode is experimentally challenging. The effective dynamics studied in this work can potentially be achieved via other qubits, e.g., by constructing Ising interactions between superconducting qubits with intrinsic T_1 protection (see Supplemental Material [54] for more details). This may lead to more amenable experimental constructions within circuit QED platforms (and potentially beyond). The photonic-Ising model can be generalized to adapt the Toom’s rule [44] or to higher dimensions [76] for a more robust perturbative stability. The full perturbative stability of the model remains an interesting open question.

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- [1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2010).
- [2] D. A. Lidar and T. A. Brun, *Quantum Error Correction* (Cambridge University Press, Cambridge, England, 2013).
- [3] D. Gottesman, Stabilizer codes and quantum error correction, Ph.D. thesis, California Institute of Technology, 1997.
- [4] S. B. Bravyi and A. Y. Kitaev, [arXiv:quant-ph/9811052](https://arxiv.org/abs/quant-ph/9811052).
- [5] A. G. Fowler, M. Mariantoni, J. M. Martinis, and A. N. Cleland, *Phys. Rev. A* **86**, 032324 (2012).
- [6] E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, *J. Math. Phys. (N.Y.)* **43**, 4452 (2002).
- [7] D. Gottesman, A. Kitaev, and J. Preskill, *Phys. Rev. A* **64**, 012310 (2001).
- [8] D. Bacon, *Phys. Rev. A* **73**, 012340 (2006).
- [9] B. Yoshida, *Ann. Phys. (Amsterdam)* **326**, 2566 (2011).
- [10] S. Roberts and S. D. Bartlett, *Phys. Rev. X* **10**, 031041 (2020).
- [11] B. M. Terhal, *Rev. Mod. Phys.* **87**, 307 (2015).
- [12] B. J. Brown, D. Loss, J. K. Pachos, C. N. Self, and J. R. Wootton, *Rev. Mod. Phys.* **88**, 045005 (2016).
- [13] H. Bombin, R. W. Chhajlany, M. Horodecki, and M. A. Martin-Delgado, *New J. Phys.* **15**, 055023 (2013).
- [14] J. Haah, *Phys. Rev. A* **83**, 042330 (2011).
- [15] S. Bravyi and J. Haah, *Phys. Rev. Lett.* **111**, 200501 (2013).
- [16] S. Chesi, B. Röthlisberger, and D. Loss, *Phys. Rev. A* **82**, 022305 (2010).
- [17] N. P. Breuckmann, K. Duivenvoorden, D. Michels, and B. M. Terhal, *Quantum Inf. Comput.* **17**, 0181 (2017).
- [18] R. Alicki, M. Fannes, and M. Horodecki, *J. Phys. A* **42**, 065303 (2009).
- [19] R. Alicki, M. Horodecki, P. Horodecki, and R. Horodecki, *Open Syst. Inf. Dyn.* **17**, 1 (2010).
- [20] Y.-J. Liu and S. Lieu, *Phys. Rev. A* **109**, 022422 (2024).
- [21] J. P. Paz and W. H. Zurek, *Proc. R. Soc. A* **454**, 355 (1998).

- [22] J. P. Barnes and W. S. Warren, *Phys. Rev. Lett.* **85**, 856 (2000).
- [23] C. Ahn, A. C. Doherty, and A. J. Landahl, *Phys. Rev. A* **65**, 042301 (2002).
- [24] M. Sarovar and G. J. Milburn, *Phys. Rev. A* **72**, 012306 (2005).
- [25] O. Oreshkov and T. A. Brun, *Phys. Rev. A* **76**, 022318 (2007).
- [26] J. Kerckhoff, H. I. Nurdin, D. S. Pavlichin, and H. Mabuchi, *Phys. Rev. Lett.* **105**, 040502 (2010).
- [27] J.-M. Lihm, K. Noh, and U. R. Fischer, *Phys. Rev. A* **98**, 012317 (2018).
- [28] F. Pastawski, L. Clemente, and J. I. Cirac, *Phys. Rev. A* **83**, 012304 (2011).
- [29] M. Mirrahimi, Z. Leghtas, V. V. Albert, S. Touzard, R. J. Schoelkopf, L. Jiang, and M. H. Devoret, *New J. Phys.* **16**, 045014 (2014).
- [30] E. Kapit, *Phys. Rev. Lett.* **116**, 150501 (2016).
- [31] F. Reiter, A. S. Sørensen, P. Zoller, and C. A. Muschik, *Nat. Commun.* **8**, 1822 (2017).
- [32] K. Fujii, M. Negoro, N. Imoto, and M. Kitagawa, *Phys. Rev. X* **4**, 041039 (2014).
- [33] N. Ofek, A. Petrenko, R. Heeres, P. Reinhold, Z. Leghtas, B. Vlastakis, Y. Liu, L. Frunzio, S. M. Girvin, L. Jiang, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf, *Nature (London)* **536**, 441 (2016).
- [34] A. Gyenis, P. S. Mundada, A. Di Paolo, T. M. Hazard, X. You, D. I. Schuster, J. Koch, A. Blais, and A. A. Houck, *PRX Quantum* **2**, 010339 (2021).
- [35] P. Campagne-Ibarcq, A. Eickbusch, S. Touzard, E. Zalgaller, N. E. Frattini, V. V. Sivak, P. Reinhold, S. Puri, S. Shankar, R. J. Schoelkopf, L. Frunzio, M. Mirrahimi, and M. H. Devoret, *Nature (London)* **584**, 368 (2020).
- [36] Z. Leghtas, S. Touzard, I. M. Pop, A. Kou, B. Vlastakis, A. Petrenko, K. M. Sliwa, A. Narla, S. Shankar, M. J. Hatridge *et al.*, *Science* **347**, 853 (2015).
- [37] R. W. Heeres, P. Reinhold, N. Ofek, L. Frunzio, L. Jiang, M. H. Devoret, and R. J. Schoelkopf, *Nat. Commun.* **8**, 94 (2017).
- [38] R. Lescanne, M. Villiers, T. Peronin, A. Sarlette, M. Delbecq, B. Huard, T. Kontos, M. Mirrahimi, and Z. Leghtas, *Nat. Phys.* **16**, 509 (2020).
- [39] C. Berdou, A. Murani, U. Reglade, W. Smith, M. Villiers, J. Palomo, M. Rosticher, A. Denis, P. Morfin, M. Delbecq *et al.*, *PRX Quantum* **4**, 020350 (2023).
- [40] S. Lieu, R. Belyansky, J. T. Young, R. Lundgren, V. V. Albert, and A. V. Gorshkov, *Phys. Rev. Lett.* **125**, 240405 (2020).
- [41] O. Shtanko, Y.-J. Liu, S. Lieu, A. V. Gorshkov, and V. V. Albert, *arXiv:2308.16233* (2023).
- [42] J. M. Gertler, B. Baker, J. Li, S. Shirol, J. Koch, and C. Wang, *Nature (London)* **590**, 243 (2021).
- [43] Z. Leghtas, G. Kirchmair, B. Vlastakis, R. J. Schoelkopf, M. H. Devoret, and M. Mirrahimi, *Phys. Rev. Lett.* **111**, 120501 (2013).
- [44] A. L. Toom, *Probl. Inf. Transm.* **10**, 239 (1974).
- [45] G. Lindblad, *Commun. Math. Phys.* **48**, 119 (1976).
- [46] If the code space spans the kernel of \mathcal{L}_r , then a natural choice for the decoder is $\mathcal{E}_r = \lim_{t \rightarrow \infty} e^{\mathcal{L}_r t}$. Such a decoder is typically used for the cat code. However, if \mathcal{L}_r has other steady states which are not in the code space, such a decoder is not ideal. We do not use this decoder for the Ising model, since its \mathcal{L}_r has steady states outside of the code space.
- [47] C. Chamberland, K. Noh, P. Arrangoiz-Arriola, E. T. Campbell, C. T. Hann, J. Iverson, H. Putterman, T. C. Bohdanowicz, S. T. Flammia, A. Keller, G. Refael, J. Preskill, L. Jiang, A. H. Safavi-Naeini, O. Painter, and F. G. S. L. Brandão, *PRX Quantum* **3**, 010329 (2022).
- [48] L. Gilles, B. M. Garraway, and P. L. Knight, *Phys. Rev. A* **49**, 2785 (1994).
- [49] We could also include jumps that flip a spin if there are two domain walls surrounding it, e.g., $L^{(2)} = \sqrt{\gamma} X_{x,y} P_{x,y;\rightarrow}^+ P_{x,y;\uparrow}^+ P_{x-1,y;\rightarrow}^- P_{x,y-1;\uparrow}^-$, since such a process does not change the energy and, hence, respects detailed balance (for any rate γ). Consequently, such jumps do not change the exact thermal steady state solutions. We choose not to include such jumps, since the mean-field solution is more accurate without them.
- [50] M. B. Plenio and P. L. Knight, *Rev. Mod. Phys.* **70**, 101 (1998).
- [51] A. J. Daley, *Adv. Phys.* **63**, 77 (2014).
- [52] B. Buča and T. Prosen, *New J. Phys.* **14**, 073007 (2012).
- [53] If the symmetry-broken initial state $|\bar{0}\rangle$ is recoverable, then so is any state in the code space. This is because an arbitrary state in the code space can be expressed as $c_0|\bar{0}\rangle + c_1|\bar{1}\rangle = (c_0 - c_1)|\bar{0}\rangle + c_1(|\bar{0}\rangle + |\bar{1}\rangle)$. Since the dynamics is linear, and the symmetric state $|\bar{0}\rangle + |\bar{1}\rangle$ can always be properly recovered in the absence of Z dephasing, this implies that any state in the code space is recoverable if $|\bar{0}\rangle$ is recoverable.
- [54] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.133.030601> for supporting calculations and a circuit-QED proposal for the realization of the photonic-Ising model, which includes Refs. [55–67].
- [55] H. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, New York, 2002).
- [56] A. Schilling, M. Cantoni, J. D. Guo, and H. R. Ott, *Nature (London)* **363**, 56 (1993).
- [57] P. Brooks, A. Kitaev, and J. Preskill, *Phys. Rev. A* **87**, 052306 (2013).
- [58] D. S. Schlegel, F. Minganti, and V. Savona, *Phys. Rev. A* **106**, 022431 (2022).
- [59] T. Hillmann and F. Quijandría, *Phys. Rev. A* **107**, 032423 (2023).
- [60] Q. Xu, G. Zheng, Y.-X. Wang, P. Zoller, A. A. Clerk, and L. Jiang, *npj Quantum Inf.* **9**, 78 (2023).
- [61] B. Yurke and D. Stoler, *Phys. Rev. Lett.* **57**, 13 (1986).
- [62] A. Grimm, N. E. Frattini, S. Puri, S. O. Mundhada, S. Touzard, M. Mirrahimi, S. M. Girvin, S. Shankar, S. Shankar, and M. H. Devoret, *Nature (London)* **584**, 205 (2020).
- [63] B. Douçot and L. B. Ioffe, *Rep. Prog. Phys.* **75**, 072001 (2012).
- [64] A. Gyenis, A. Di Paolo, J. Koch, A. Blais, A. A. Houck, and D. I. Schuster, *PRX Quantum* **2**, 030101 (2021).
- [65] A. Kubica and M. E. Beverland, *Phys. Rev. A* **91**, 032330 (2015).

- [66] H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, and H. P. Büchler, *Nat. Phys.* **6**, 382 (2010).
- [67] J. T. Barreiro, M. Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller, and R. Blatt, *Nature (London)* **470**, 486 (2011).
- [68] S. Bravyi and B. Terhal, *New J. Phys.* **11**, 043029 (2009).
- [69] J. Guillaud and M. Mirrahimi, *Phys. Rev. X* **9**, 041053 (2019).
- [70] J. Cohen, Autonomous quantum error correction with superconducting qubits, Ph.D. thesis, Ecole Normale Supérieure, 2017.
- [71] J. Cohen, W. C. Smith, M. H. Devoret, and M. Mirrahimi, *Phys. Rev. Lett.* **119**, 060503 (2017).
- [72] L. E. Thomas, *Commun. Math. Phys.* **126**, 1 (1989).
- [73] R. H. Schonmann, *Commun. Math. Phys.* **112**, 409 (1987).
- [74] D. Randall, in *Proceedings of the Seventeenth Annual ACM-SIAM Symposium on Discrete Algorithm, SODA '06* (Society for Industrial and Applied Mathematics, USA, 2006), p. 870–879.
- [75] J. P. Home, D. Hanneke, J. D. Jost, D. Leibfried, and D. J. Wineland, *New J. Phys.* **13**, 073026 (2011).
- [76] D. Poulin, R. G. Melko, and M. B. Hastings, *Phys. Rev. B* **99**, 094103 (2019).